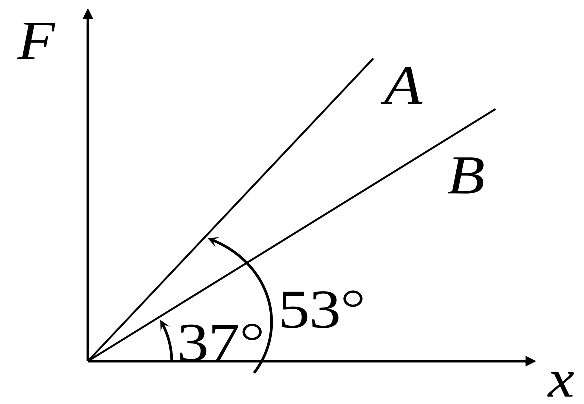
1.如圖為A、B兩彈簧受力F與伸長量x之關係曲線，則當二彈簧（1）串聯（2）並聯 後，$F-x$ 圖之斜率各為何？

[解析]



$$K_A = \tan 53°$$

$$K_B = \tan 37°$$

$$(1)\text{串聯}\ \frac{1}{K} = \frac{1}{K_A} + \frac{1}{K_B} = \frac{1}{\tan 53°} + \frac{1}{\tan 37°} = \frac{3}{4} + \frac{4}{3} = \frac{25}{12} \rightarrow K = \frac{12}{25}$$

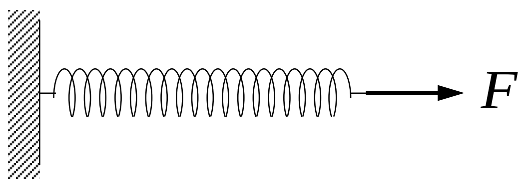
$$(2)\text{並聯}\ K = K_A + K_B = \tan 37° + \tan 53° = \frac{4}{3} + \frac{3}{4} = \frac{25}{12}$$

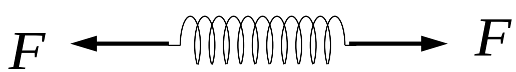
第80頁

*2.*一輕彈簧原長10*cm*水平放置一端固定在牆壁一端施以20*gw*拉力彈簧長度為20*cm*，若將彈黃剪成原來的 $\frac{1}{3}$ 長度的新彈簧，試問：

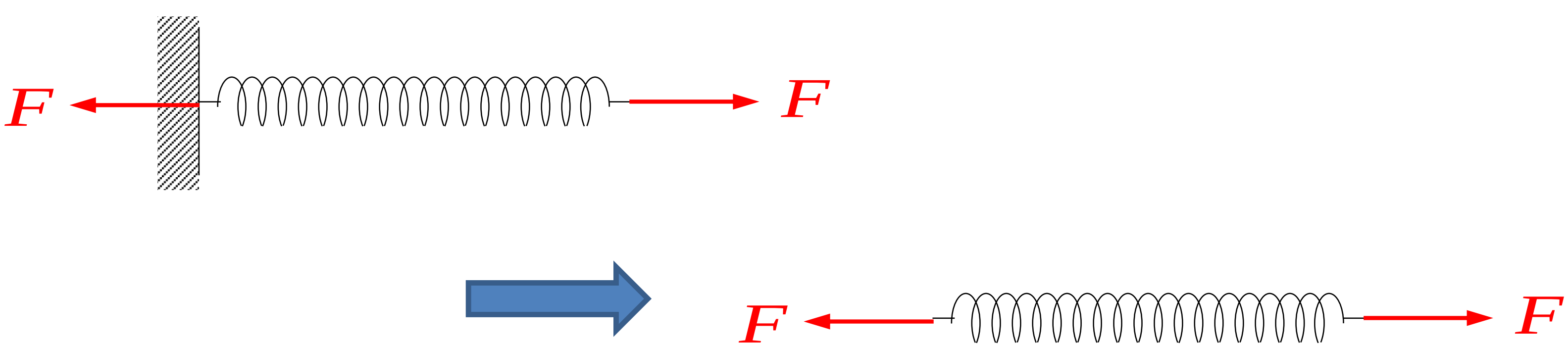
（1）兩端同施以20gw於新彈簧其伸長量為何？

（2）兩端各施以10gw、20gw於新彈簧，其伸長量為何？

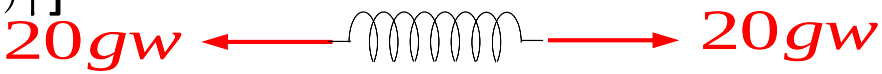


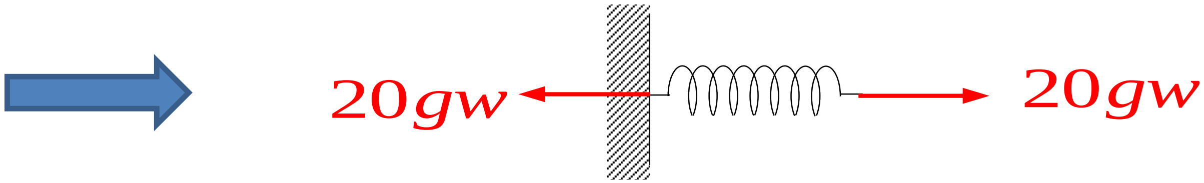


[解析]



[解析]





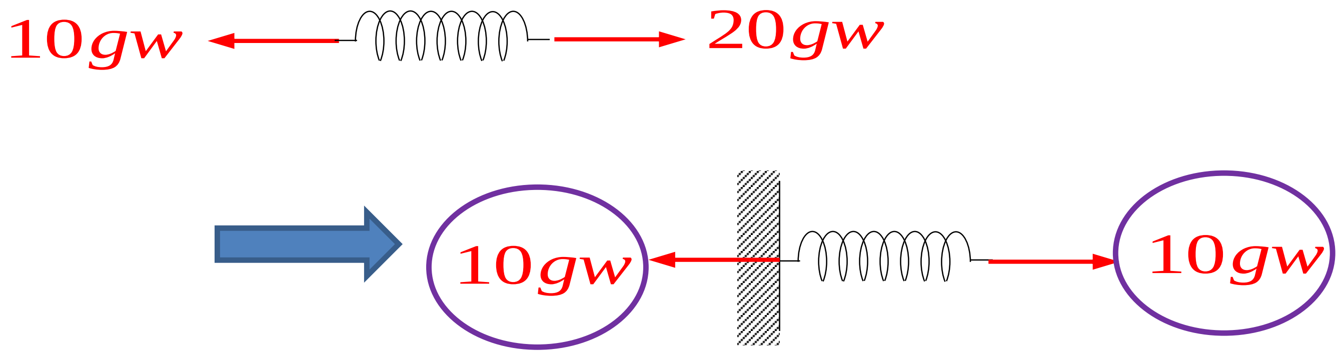
$$(1)\text{令原來彈簧彈力常數}k=\frac{20}{20-10}=2[gw/cm]$$

$$\text{則新彈簧彈力常數}k'=\frac{k}{\frac{1}{3}}=6[gw/cm]$$

$$\text{令新彈簧形變量}x'$$

$$[F_S=kx]\quad 20=6x' \quad \therefore x'=\frac{10}{3}[cm]$$

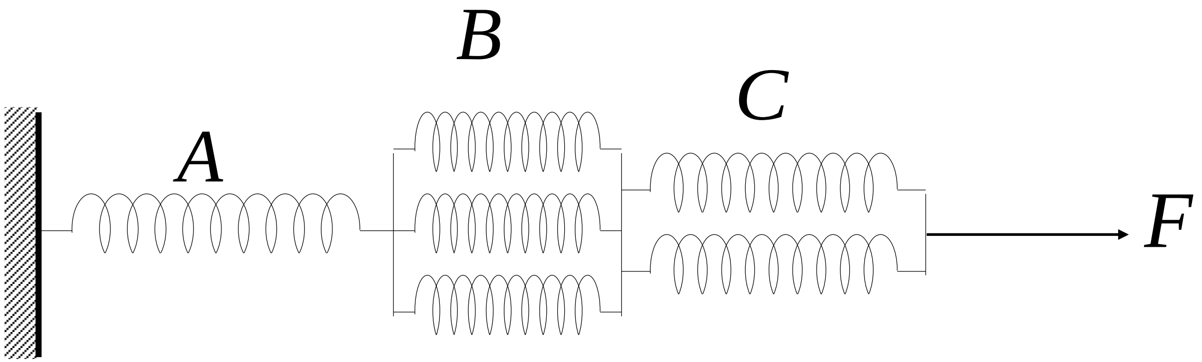
[解析]



(2)令形變量x''

$$[F_S = kx]\ \ 10 = 6x'' \quad \therefore x'' = \frac{5}{3}[cm]$$

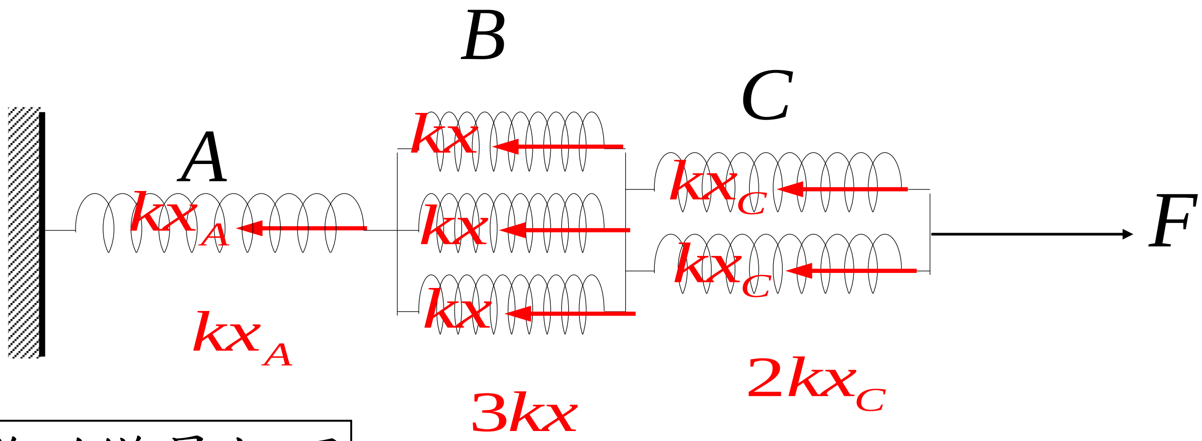
3. 圖中所示為輕質的相同彈簧組合，若*B*組伸長量為*x*，則整體的伸長量為？



並聯形變量相同

串聯彈力相同

[解析]



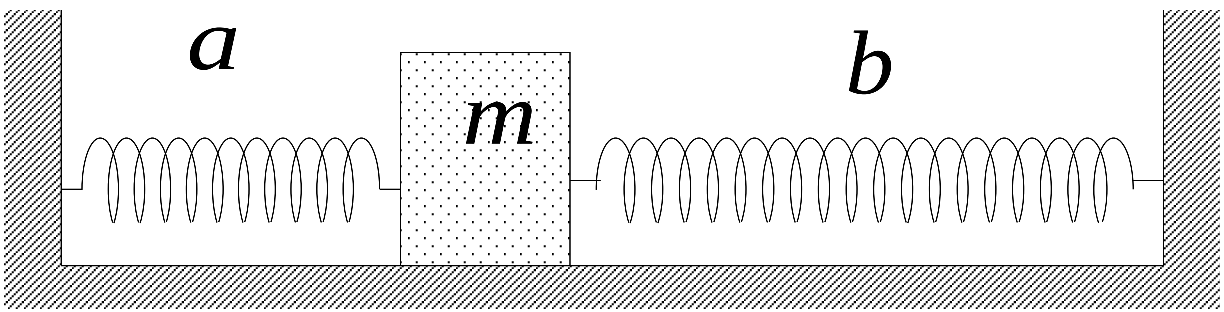
並聯形變量相同
串聯彈力相同

令A形變量x_A C形變量x_C

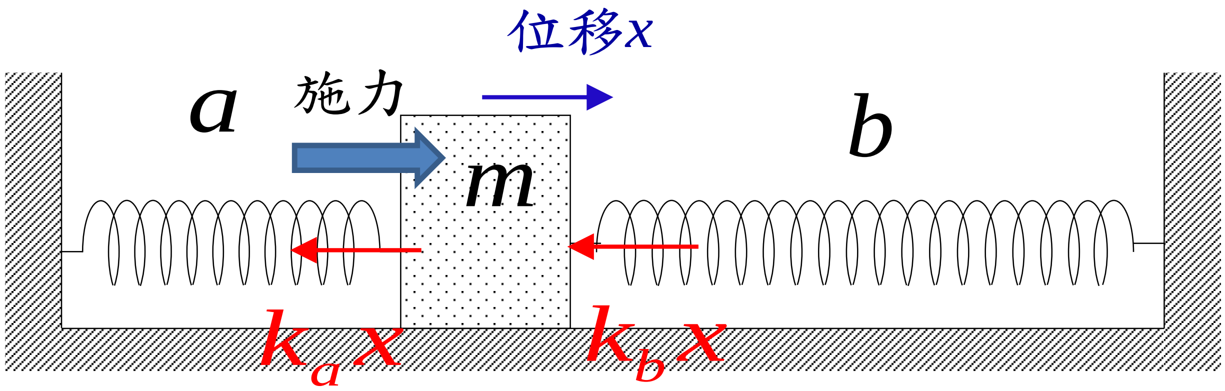
$\because$串聯$\therefore kx_A = 3kx = 2kx_C = F$ $\quad \therefore x_A = 3x \quad x_C = \dfrac{3}{2}x$

$$\therefore x_A + x + x_C = 3x + x + \frac{3}{2}x = \frac{11}{2}x$$

1.長 ，彈簧力常數 之彈簧分成分成 $\frac{\ell}{3}$ 、$\frac{2\ell}{3}$ 之***a***、***b***
兩段聯結如圖，求將***m***向右移***x***距離需施力若干？（摩擦力不計）



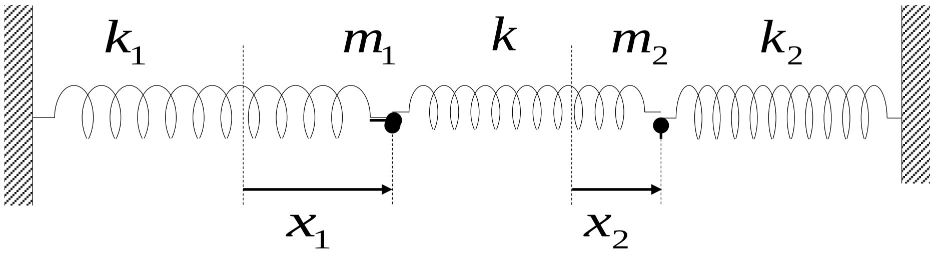
[解析]



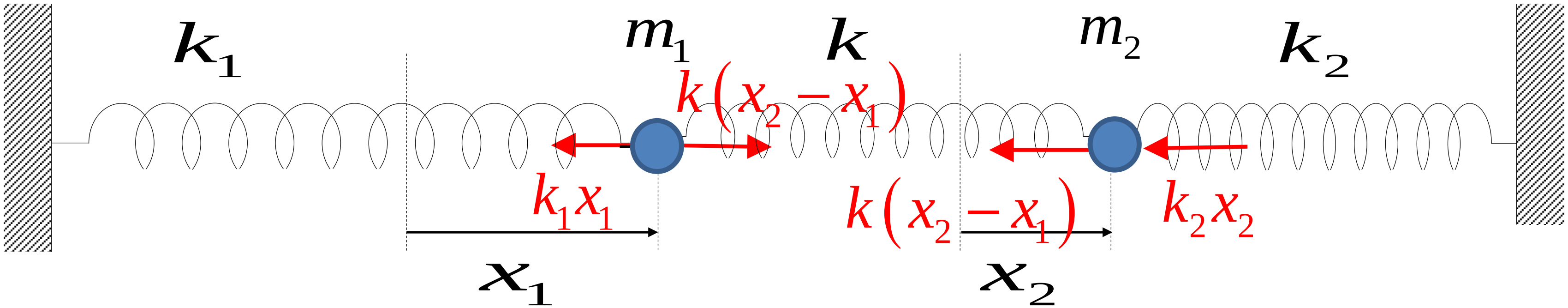
$$a\ 彈力常數 k_a = \frac{k}{\frac{1}{3}} = 3k \quad b\ 彈力常數 k_b = \frac{k}{\frac{2}{3}} = \frac{3}{2}k$$

$$施力 = k_a x + k_b x = 3kx + \frac{3}{2}kx = \frac{9}{2}kx$$

2. 二物體質量分別為m_1及m_2，以彈力常數分別為k_1，k，k_2之三個彈簧連繫起來，如圖所示。在不考 慮重力及摩擦力的情況下，設於m_1及m_2物體偏離其平衡點之位移分別為x_1及x_2（設向右位移時x為正），則m_1物體此時所受淨力為？



[解析]



已知k_1彈簧伸長x_1　k_2彈簧壓縮x_2

令k彈簧伸長$(x_{2-}x_1)$

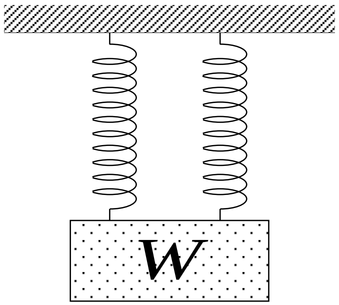
m_1 : 合力(淨力)$F_1 = k(x_2 - x_1) - k_1x_1$

m_2 : 合力(淨力)$F_2 = -k(x_2 - x_1) - k_2x_2$

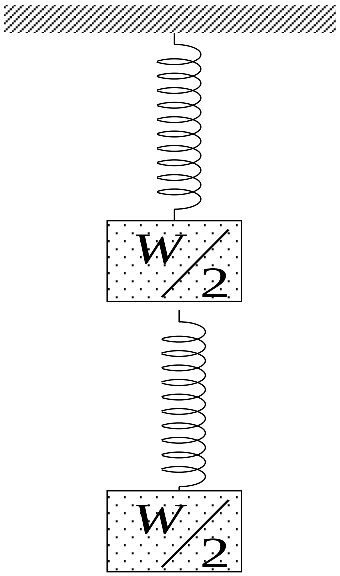
將一物重***W***懸吊於質量可忽略的彈簧下，在鉛垂直方向成平衡，此時彈簧的伸長量為***x***。今將此彈簧由中間剪斷成兩段彈簧，利用被剪斷後的二彈簧將同一重物吊起，在鉛垂方向成平衡 ，試問

（**1**）如圖（一）所示，若懸吊連接所耗去的長度可以忽略，則彈簧被重物拉長了？

（**2**）如圖（二）所示，若懸吊連接所耗去的長度可以忽略，則彈簧被重物拉長了？



(一)

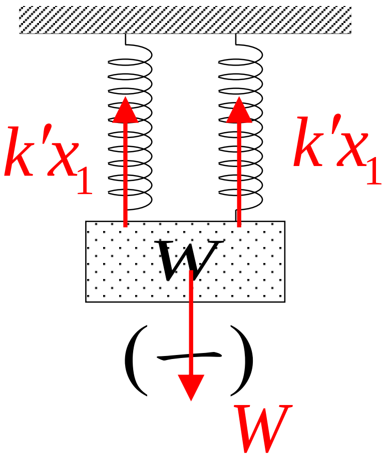


(二)

[解析]

令原來彈簧彈力常數$k=\frac{W}{x}$

則新彈簧彈力常數$k'=\frac{k}{\frac{1}{2}}=\frac{2W}{x}$



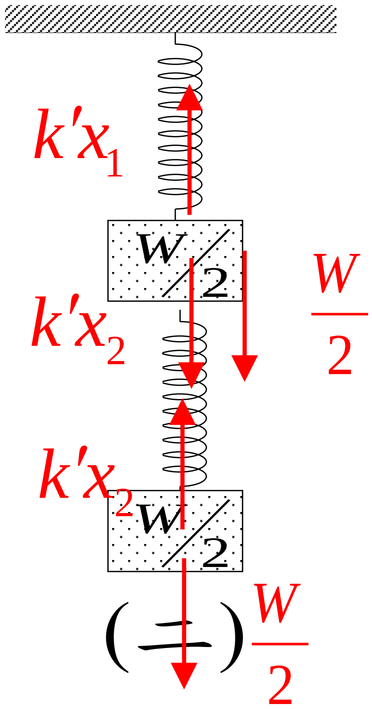
(1)令形變量x_1

物:合力=0 $2k'x_1=W\rightarrow 2\frac{2W}{x}x_1=W\ \therefore x_1=\frac{x}{4}$

[解析]

(2)令上彈簧形變量x_1　下彈簧形變量x_2

$$\begin{cases} 上物:合力=0 \quad k'x_1 = k'x_2 + \dfrac{W}{2} \cdots\cdots ① \\ 下物:合力=0 \quad k'x_2 = \dfrac{W}{2} \cdots\cdots ② \end{cases}$$

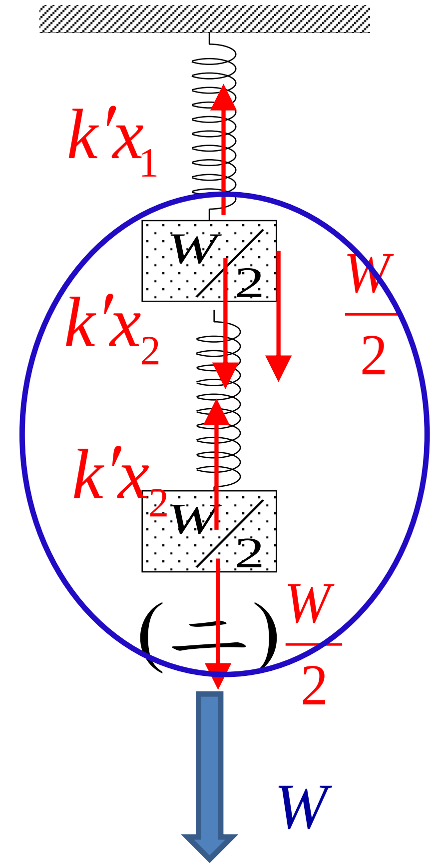


由②知　$k'x_2 = \dfrac{W}{2} \rightarrow \dfrac{2W}{x} x_2 = \dfrac{W}{2} \therefore x_2 = \dfrac{x}{4}$

②帶入①　$k'x_1 = k'x_2 + \dfrac{W}{2} \rightarrow k'x_1 = W \rightarrow \dfrac{2W}{x} x_1 = W \therefore x_1 = \dfrac{x}{2}$

第79頁

[另解]



(2)令上彈簧形變量x_1　下彈簧形變量x_2

$$\begin{cases} \text{上物+下物：合力=0} \quad k'x_1 = W \rightarrow \dfrac{2W}{x}x_1 = W \therefore x_1 = \dfrac{x}{2} \\ \text{下物：合力=0} \quad k'x_2 = \dfrac{W}{2} \rightarrow \dfrac{2W}{x}x_2 = \dfrac{W}{2} \therefore x_2 = \dfrac{x}{4} \end{cases}$$

[挑戰題]

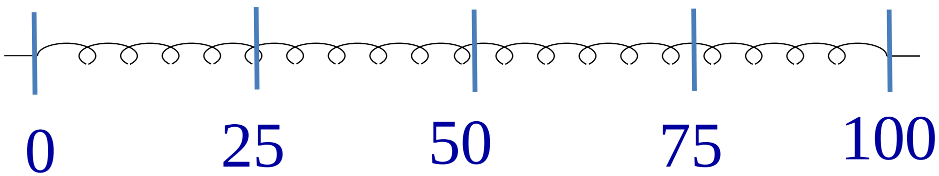
一均勻彈簧水平放置，未受力時長度為*1*米，在其上依次以*1*厘米之間隔畫上刻度，*0*，*1*， *2*，*3*，...，*99*，*100*。設此彈簧遵守虎克定律。今將此彈簧*0*刻度的一端掛在天花板上，令另一端自然下垂，平衡時測得刻度*50*與*51*相距*1.1*厘米，則

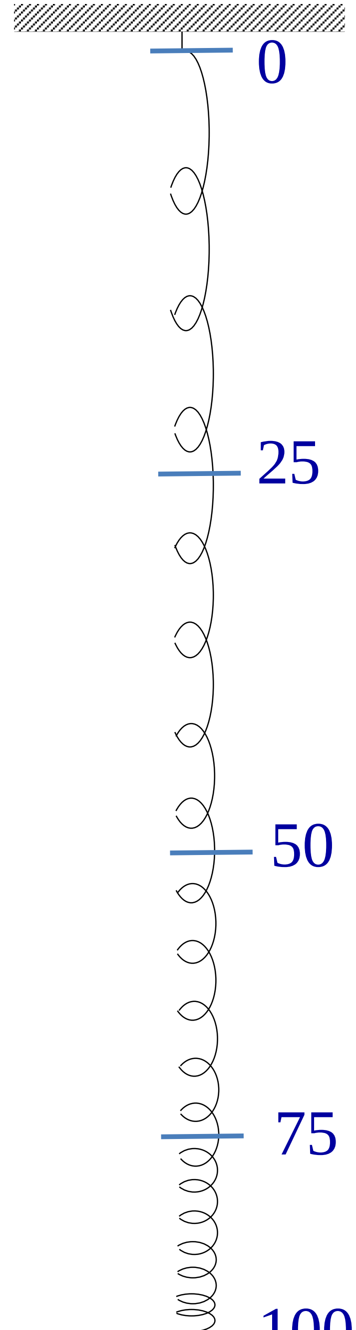
（1）下列各項兩刻度間距離最接近*2.1*厘米的是

（A）*0*與*2*（B）*24*與*26*（C）*49*與*51*（D）*74*與*76*。

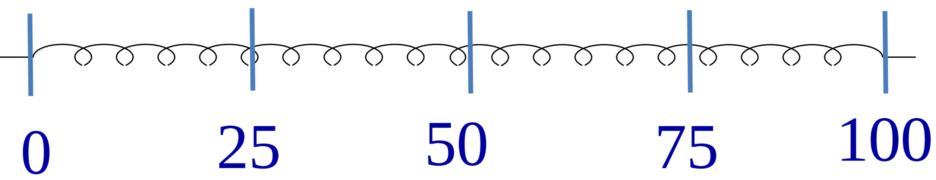
（2）此時彈簧全長約為？

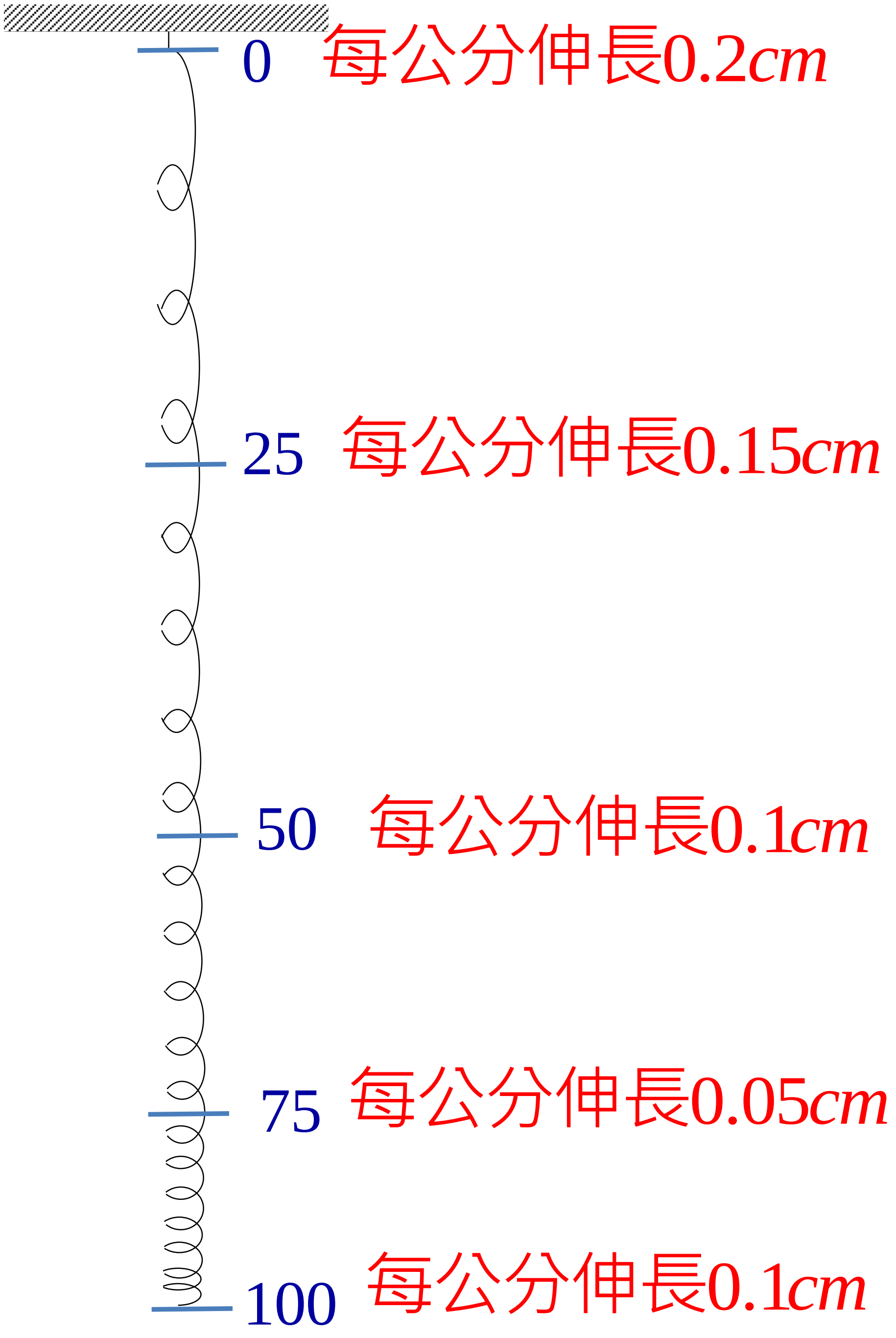
[挑戰題]





[挑戰題]


0
25
50
75
100

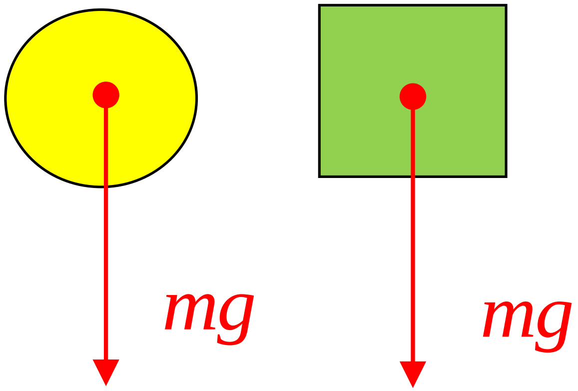

0
每公分伸長0.2cm
25
每公分伸長0.15cm
50
每公分伸長0.1cm
75
每公分伸長0.05cm
100
每公分伸長0.1cm

1. 重力 *W*

重力的大小： $W=mg$

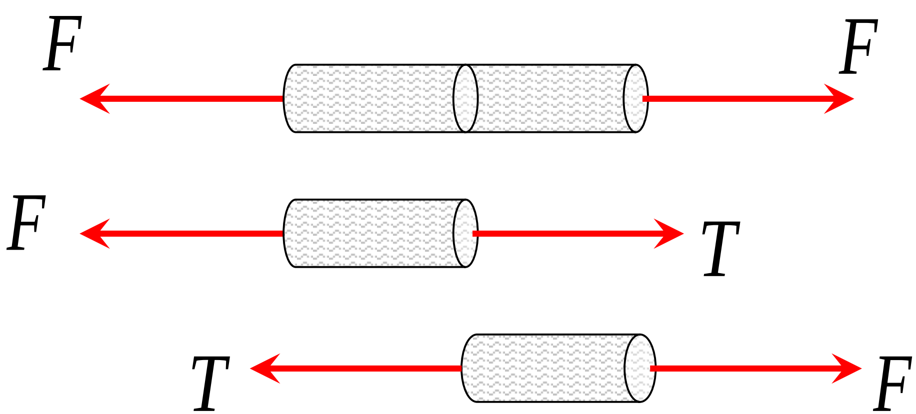
重力的方向： 鉛直向下

重力的作用點：重心（大部分材質均勻的物體位於幾何中心）



2. 繩子的張力 *T*

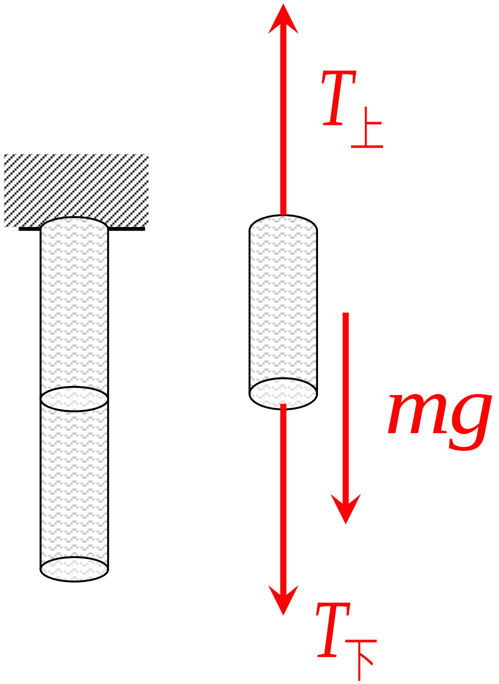
繩子的張力指向**繩子縮回的方向** 。
張力的大小只能由其它的力來決定。
繩重不計的輕繩，**同一繩子**的張力各處均相同。



看繩子的任何一部分
∵ 移動平衡 合力=0
∴ 兩邊作用力必相等

<u>若需考慮繩重</u>，則繩子的張力處處不同。

①鉛直懸吊的重繩越上端張力越大

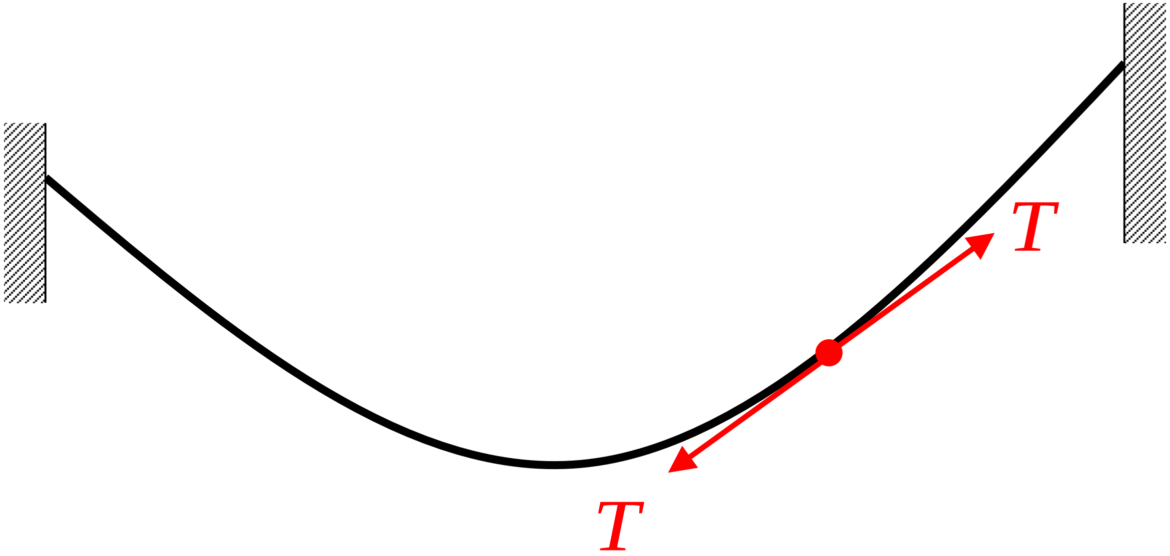


$$\because T_上 = mg + T_下$$

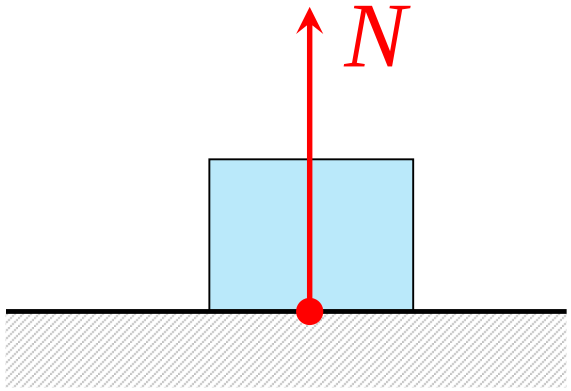
$$\therefore T_上 > T_下$$

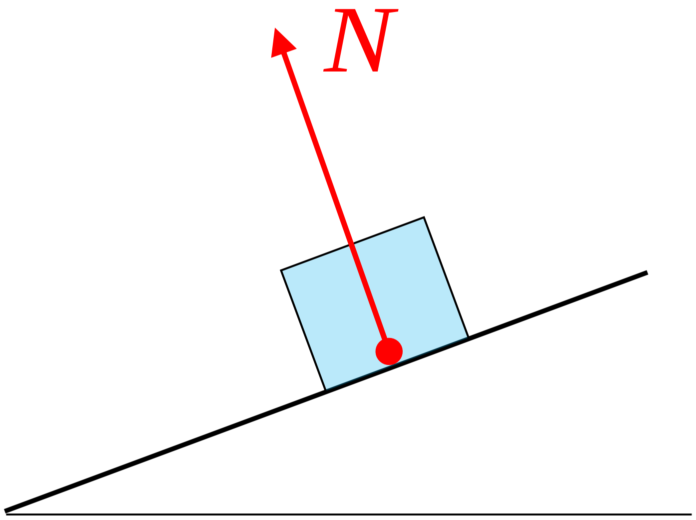
<u>若需考慮繩重</u>，則繩子的張力處處不同。

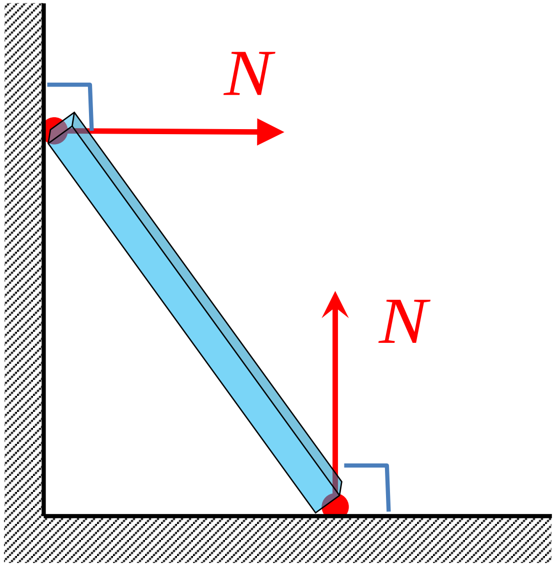
②重繩若彎曲，則張力方向為切線方向。

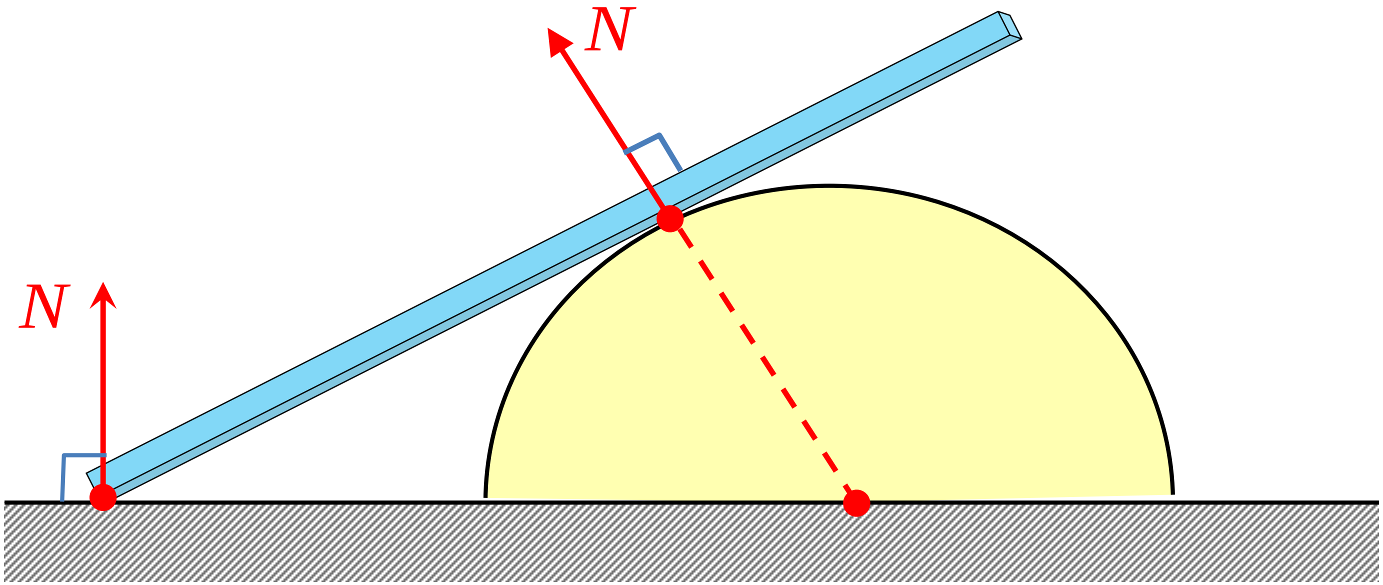


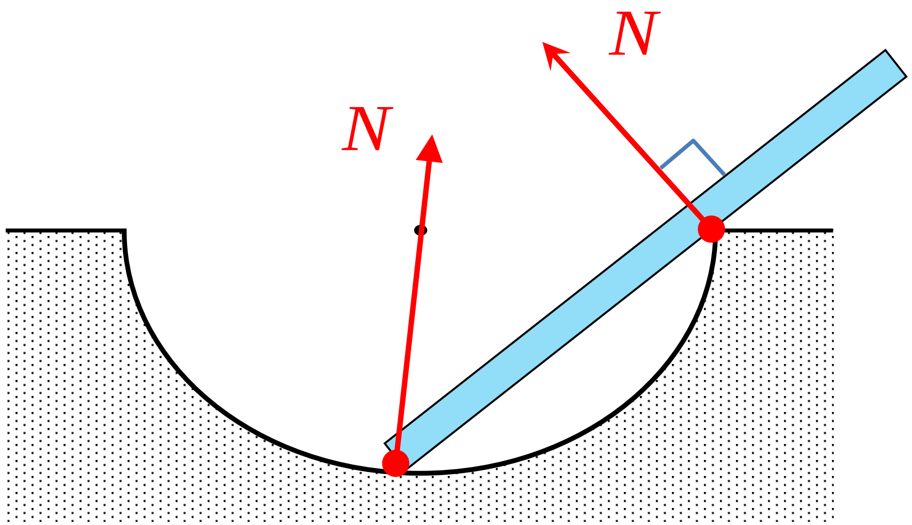
3. 正向力 *N*
（兩個接觸物體在**垂直**於接觸面方向的相互作用力）
正向力為接觸力，沒有接觸就沒有正向力
若接觸面為圓弧，則正向力的方向為沿圓弧的半徑方向

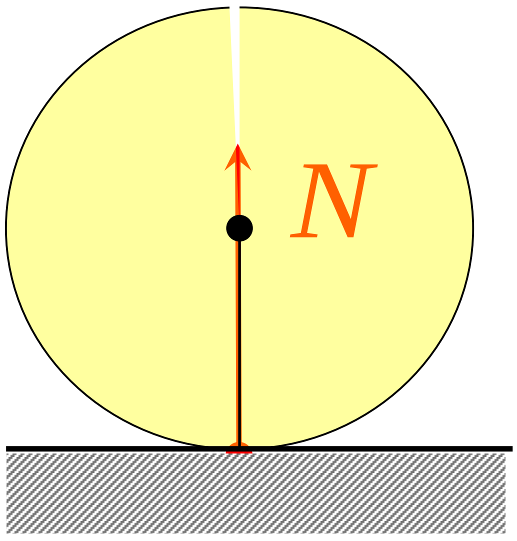





N
N


N
N


N
N

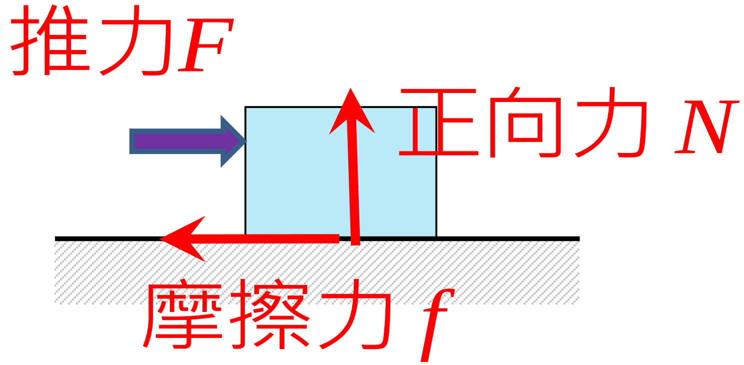

N

4. 摩擦力 f

（兩個接觸物體在<u>平行</u>於接觸面方向的相互作用力）

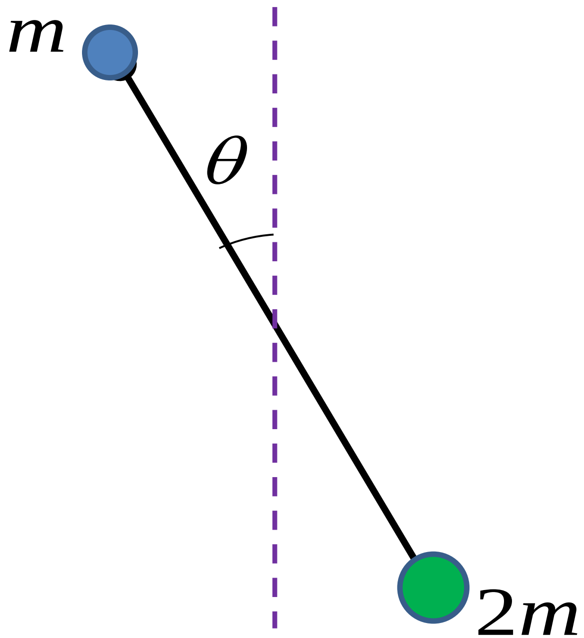
（1）方向：與相對運動或欲相對運動的方向相反。

（2）種類：靜摩擦力f_S、動摩擦力f_k。

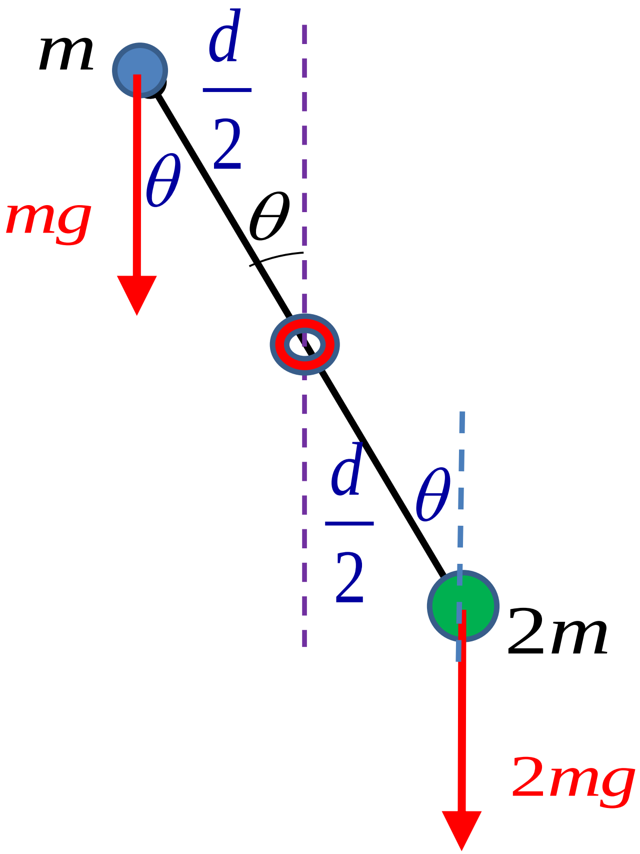


摩擦力種類	運動狀態	固體間的摩擦力公式
靜摩擦力 f_S	靜止	$f_S = F$（平行接觸面外力）
最大靜摩擦力 $f_S(max)$	靜止但恰要滑動	$f_S(max) = \mu_S N$（正向力）
動摩擦力 f_k	滑動	$f_k = \mu_k N$

*2.*一長度為d，質量可忽略不計的細桿，其中心點O固定，兩端各置有質量為m及$2m$的質點，細桿與鉛垂方向夾角為θ，設重力加速度為g，則重力對O點所產生的力矩為若干？



[解析]



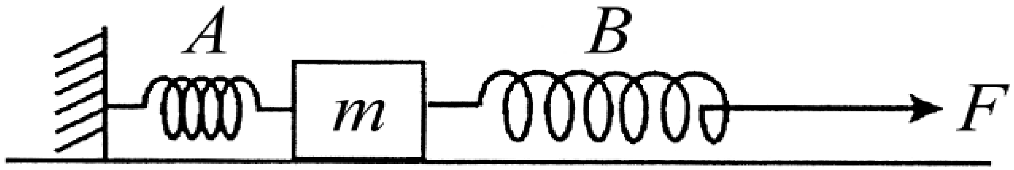
$$\boxed{\tau = rF\sin\theta = d\cdot F}$$

令順時針力矩為正

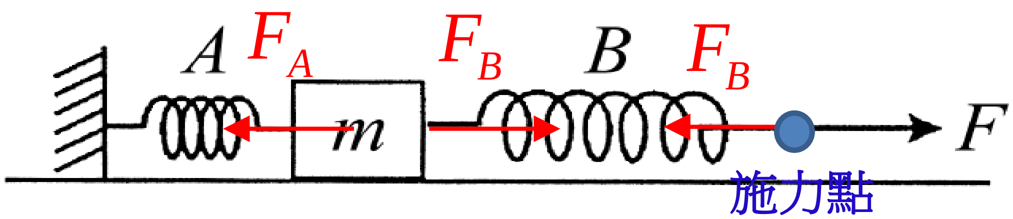
$$\frac{d}{2}\times 2mg\times\sin\theta-\frac{d}{2}\times mg\times\sin\theta=\frac{mgd}{2}\sin\theta$$

順時針

1. 將力常數為 ***6N/cm***，長 ***40cm*** 的彈簧分割成 ***A***、***B*** 兩部份，長度比為 ***1：3***，如圖示，今施力 ***F*** 使 ***m*** 的物體向右移 ***1cm***，則 ***F*** 的值多大？ ***B*** 彈簧伸長多少？



[解析]



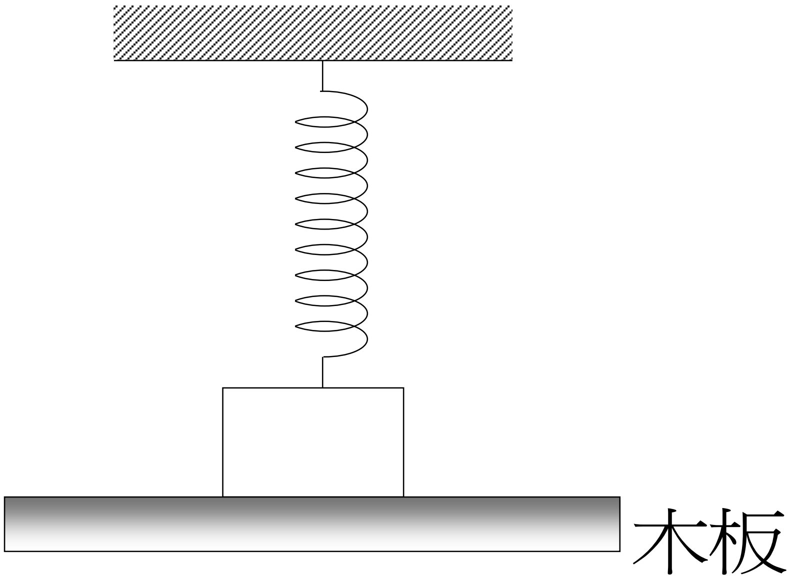
$$A\text{ 彈力常數}k_A=\frac{6}{\frac{1}{4}}=24\quad B\text{ 彈力常數}k_B=\frac{6}{\frac{3}{4}}=8$$

$$\text{已知}A\text{形變量}x_A=1cm\quad \text{令}B\text{形變量}x_B$$

$$m:\text{合力}=0\quad F_A=F_B\quad 24\times1=8\times x_B\ \therefore x_B=3[cm]$$

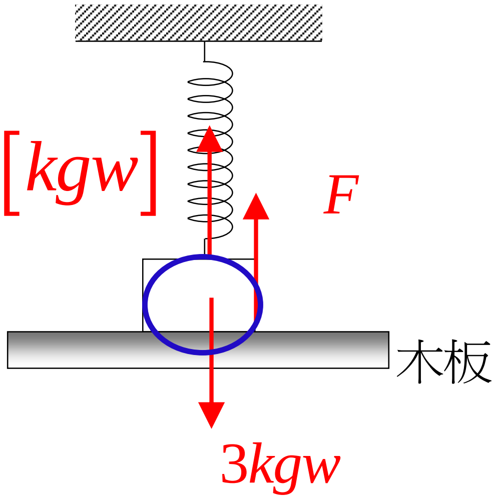
$$\text{施力點}:\text{合力}=0\quad F=F_B=8\times3=24[N]$$

2.一彈簧長*20 cm*，彈力常數為*10 kgw/m*，一端懸於天花板，另一端懸掛*3 kgw*之重錘，如圖所示，當系統達平衡時，彈簧長為*30 cm*，則木板作用於重錘之力大小為多少？



[解析]

$$kx=10\times\left(\frac{20-10}{100}\right)=1\left[kgw\right]$$



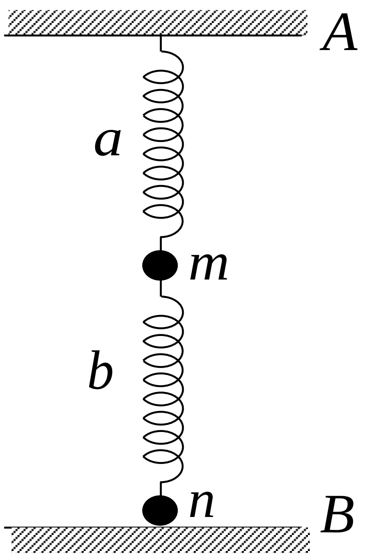
令木板作用力F

重錘:合力$=0$ $1+F=3$ $\therefore F=2\left[kgw\right]$

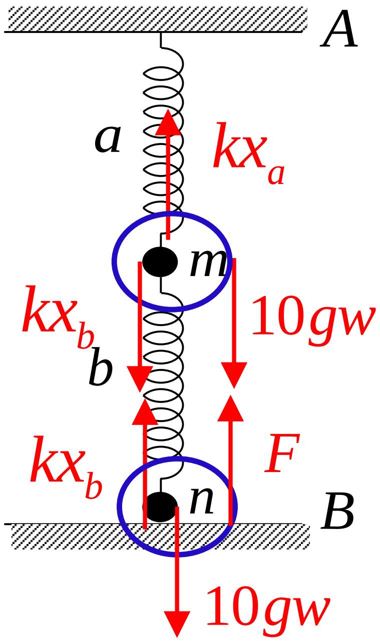
如圖，長*20* cm，力常數$k = 2.0$ gw / cm之完全相同彈簧兩條*a*、*b*（質量不計），重量*10* gw之重物*m*、*n*兩個（大小不計），如圖連接，彈簧*a*之上端固定於*A*，重物*n*放在平臺上，全體保持鉛直，試問：

（1） 平臺由靜止緩緩下移使*AB*之距離為*50* cm，則*a*彈簧長為cm？

（2）承上題，此時平臺對*n* 之作用力為gw



[解析]



(1)令a彈簧伸長x_a　b彈簧伸長x_b

$$\begin{cases} x_a + x_b = 50 - 40 = 10 \cdots\cdots ① \\ m: 合力 = 0 \quad kx_a = 10 + kx_b \rightarrow 2x_a = 10 + 2x_b \rightarrow x_a = 5 + x_b \cdots\cdots ② \end{cases}$$

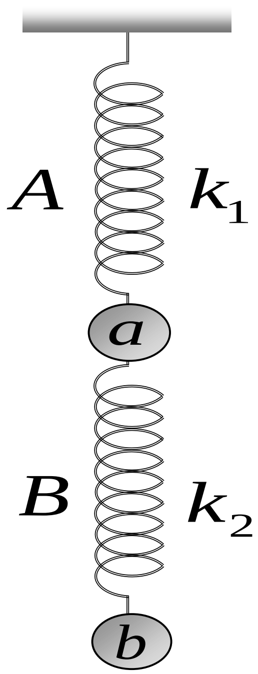
②帶入①得　$x_b = \dfrac{5}{2} \quad x_a = \dfrac{15}{2} \rightarrow a彈簧長 = 20 + x_a = 27.5$

(2)令平台對n作用力F

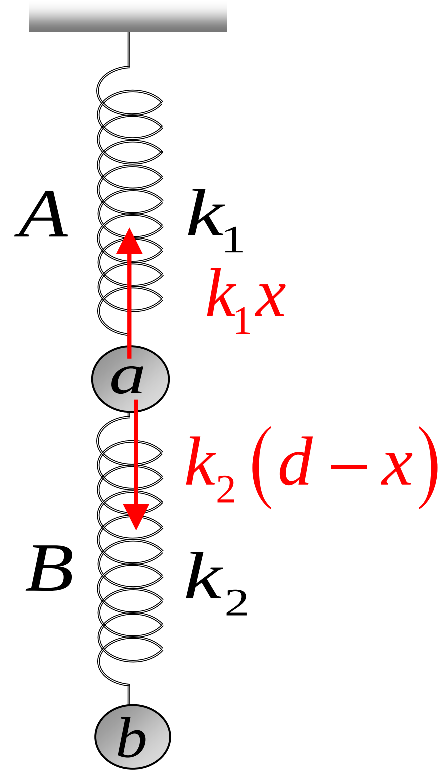
$$n: 合力 = 0 \quad kx_b + F = 10 \rightarrow 2 \times \frac{5}{2} + F = 10 \rightarrow F = 5$$

[挑戰題]

彈力常數分別為k_1、k_2之兩彈簧，與質量分別為m_1、m_2之a、b兩小球加掛如圖達成平衡，若平衡後使b球位置由平衡下降d，則a球之位置自原平衡處下降若干。



[解析]



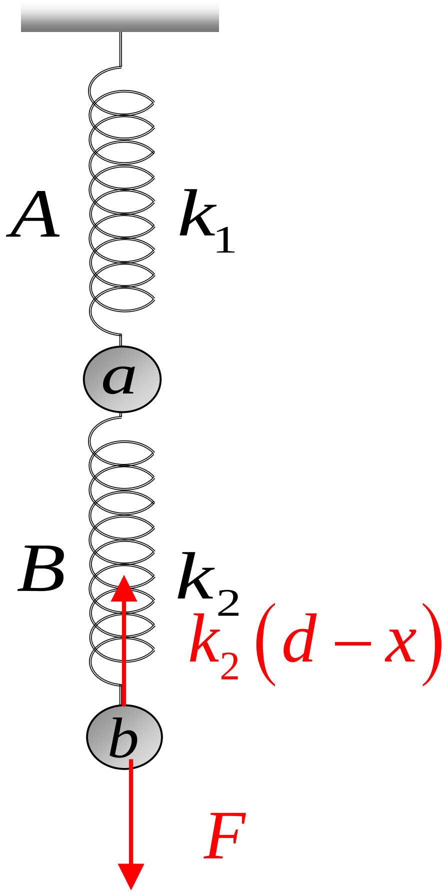
已知b球下降d

令a 球下降x

則A彈簧再伸長x B彈簧再伸長$d-x$

a：合力=0 $k_2(d-x)=k_1 x \rightarrow x=\dfrac{k_2 d}{k_1+k_2}$

[解析]

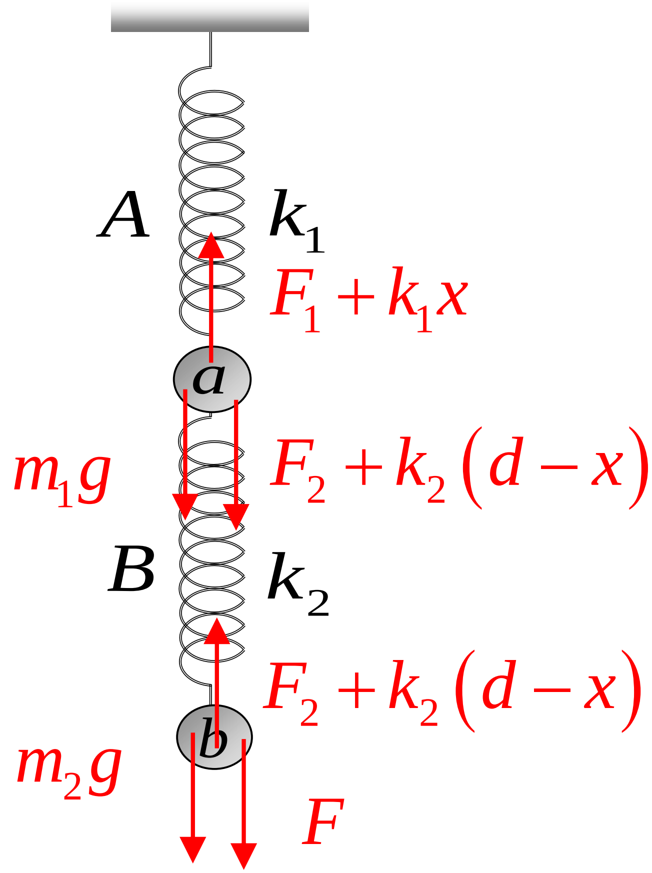


b:合力=0 $k_2(d-x)=F \rightarrow F=\dfrac{k_1k_2}{k_1+k_2}d$

[另解]

已知b球下降d

令a 球下降x

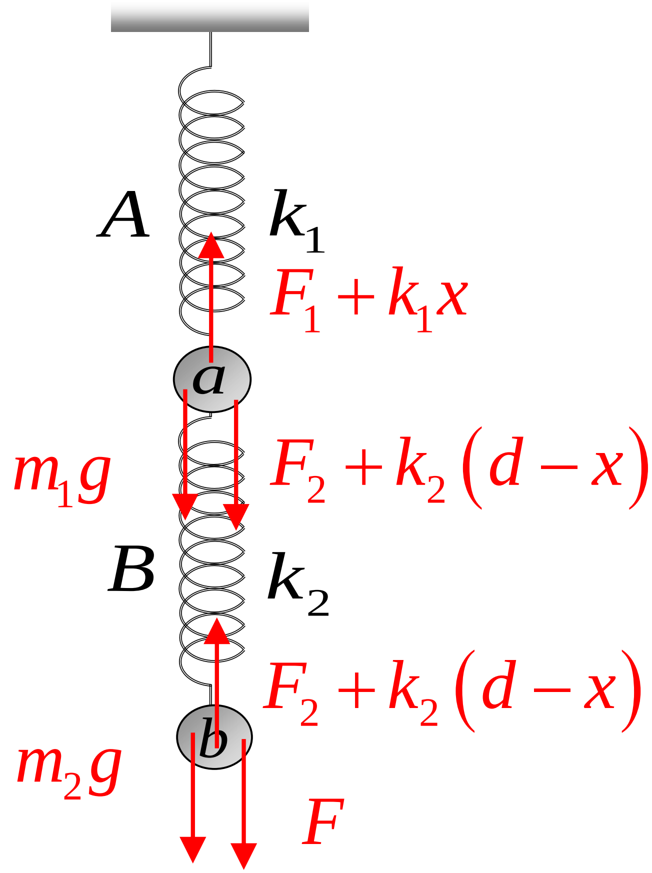


則A彈簧再伸長x　B彈簧再伸長$d-x$

原本平衡 a:合力$=0$　$F_1=F_2+m_1g$

新平衡 a:合力=0　$F_1+k_1x=F_2+m_1g+k_2(d-x)\rightarrow x=\dfrac{k_2d}{k_1+k_2}$

[另解]



已知b球下降d

令a 球下降x

則A彈簧再伸長x B彈簧再伸長$d-x$

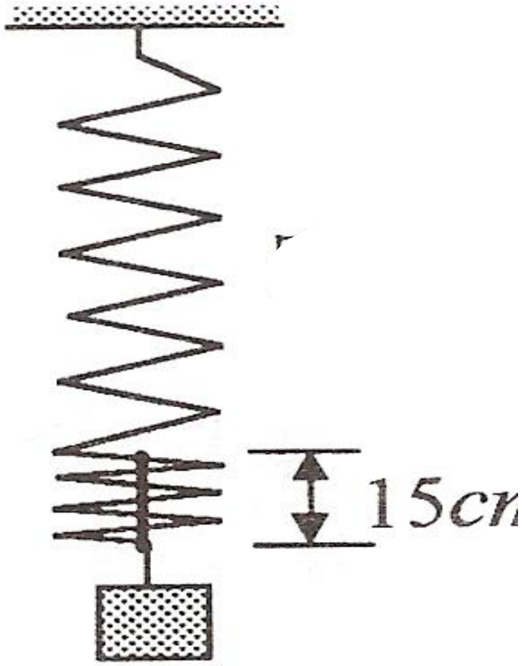
原本平衡 b:合力$=0$ $F_2=m_2g$

新平衡 a:合力=0 $F_2+k_2(d-x)=F+m_2g$

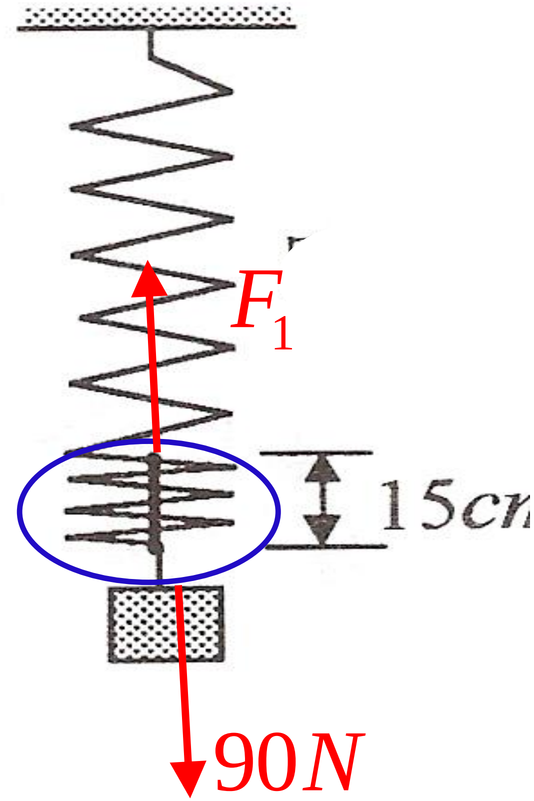
$$\rightarrow F=k_2(d-x)=\frac{k_1k_2}{k_1+k_2}d$$

[挑戰題]

如圖所示，一鉛直懸掛的輕彈簧下端掛有一重量為90N 的物體。已知彈簧的自然長度為60cm，力常數為600N / m，今將彈簧底部占三分之一全長的部分，以一15cm 的輕繩兩端分別扣住，使此部份的彈簧長度不能再行伸長，則在此情況下，（1）彈簧的全長為？（2）繩子的張力為？



[解析]



(1)

$\frac{2}{3}$長度彈簧的彈力常數$k_1=600\times\frac{3}{2}=900[N/m]$

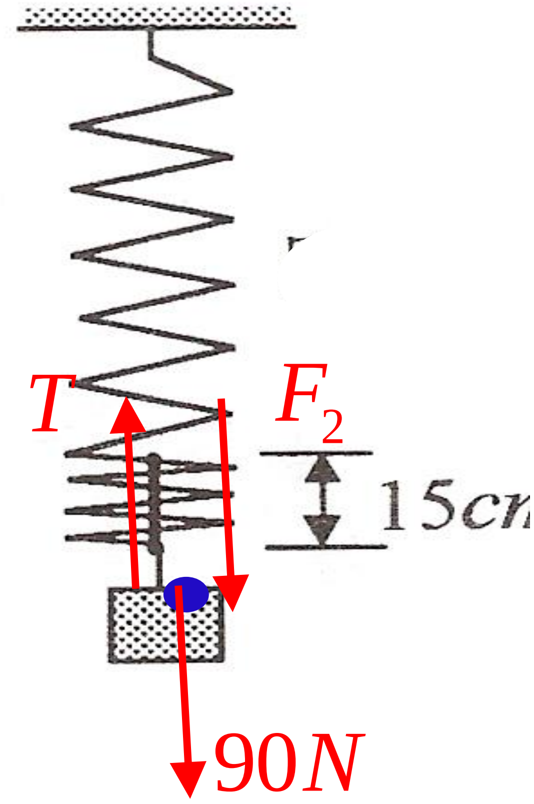
令$\frac{2}{3}$長度彈簧平衡伸長量x_1　　則彈力$F_1=k_1x_1=900x_1$

被繩綁住彈簧與繩：合力$=0$　$F_1=90\rightarrow 900x_1=90$

$$\therefore x_1=0.1[m]=10[cm]$$

總長度$=60\times\frac{2}{3}+10+15=65[cm]$

[解析]



(2)

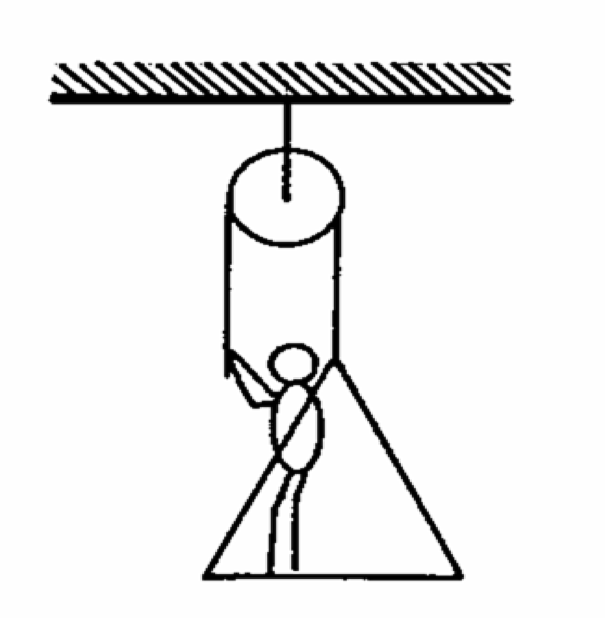
$\frac{1}{3}$長度彈簧的彈力常數$k_2=600\times\frac{3}{1}=1800[N/m]$

則彈力$F_2=k_2x_2=1800\times(0.2-0.15)=90[N]$

令繩張力T

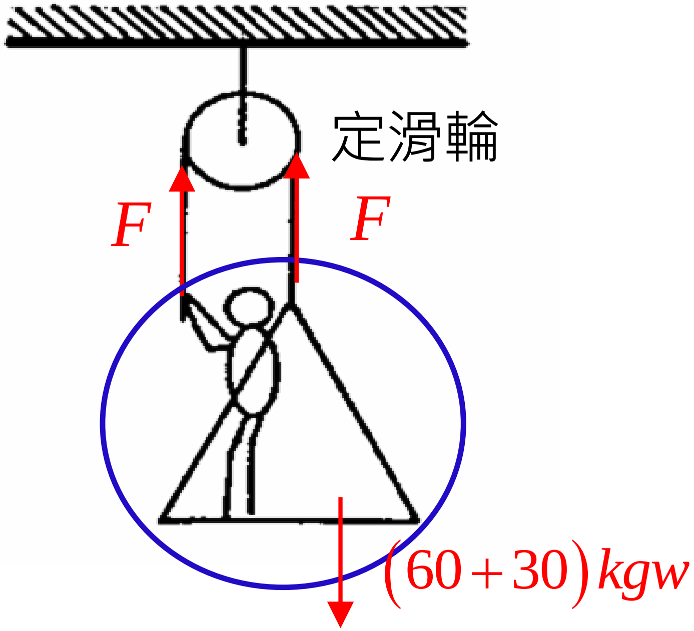
繩下端點：合力$=0$ $T=F_2+90=180[N]$

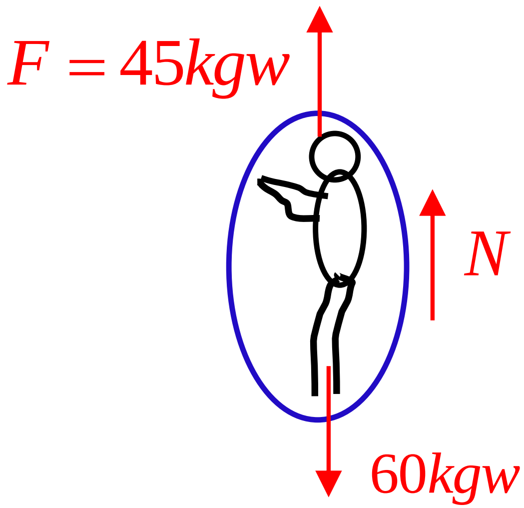
*1.(1)*一人重*60*公斤站在一重*30*公斤之平台上，垂直拉下一繞過滑輪之繩索 (如圖) ，滑論重15公斤重。設滑輪及繩索之摩擦與繩索質量可略而不計，則此人至少要施力多少公斤重始能將平台拉起？



(一)

[解析]





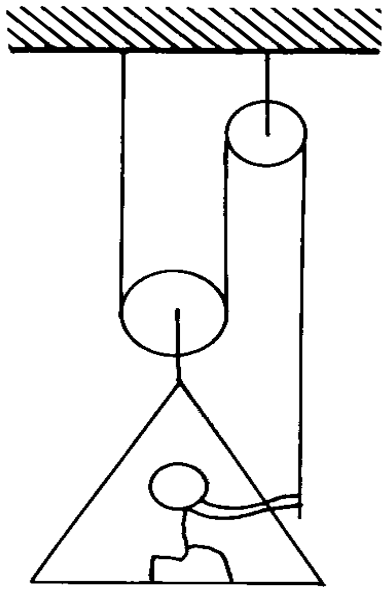
令手施力F　手的拉力=繩子張力

人+平台:合力$=0$　$2F=90 \rightarrow F=45[kgw]$

人:合力$=0$　$45+N=60 \rightarrow N=15[kgw]$

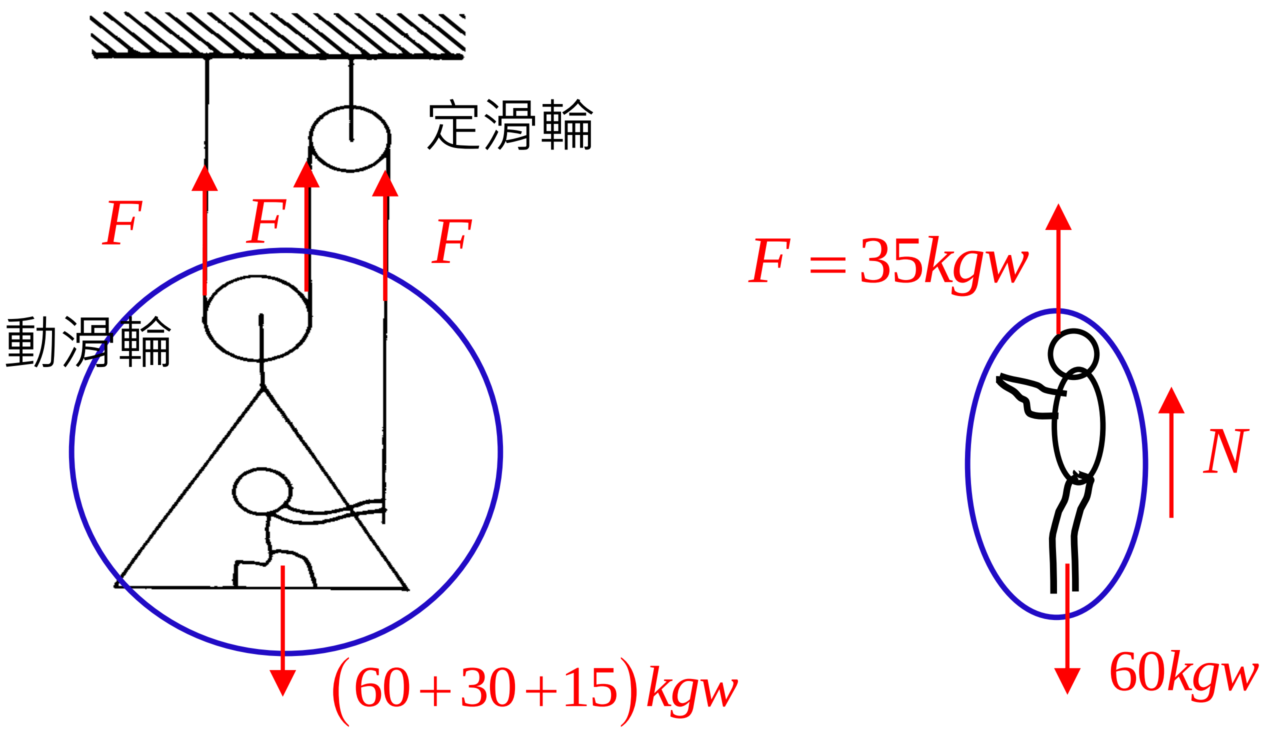
第97頁

*1.(2)*一人重*60*公斤重站在一重30公斤重之平台上，垂直拉下一繞過滑輪之繩索，每個滑論重15公斤重，如圖，設滑輪及繩索之摩擦與繩索質量可略而不計，則此人至少要施力多少公斤重始能將平台拉起？



(二)

[解析]



令手施力F　手的拉力=繩子張力

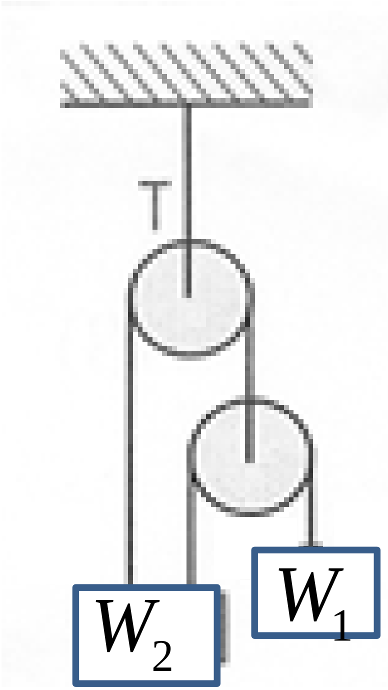
人+平台+動滑輪:合力$=0$　$3F=105 \rightarrow F=35[kgw]$

人:合力$=0$　$35+N=60 \rightarrow N=25[kgw]$

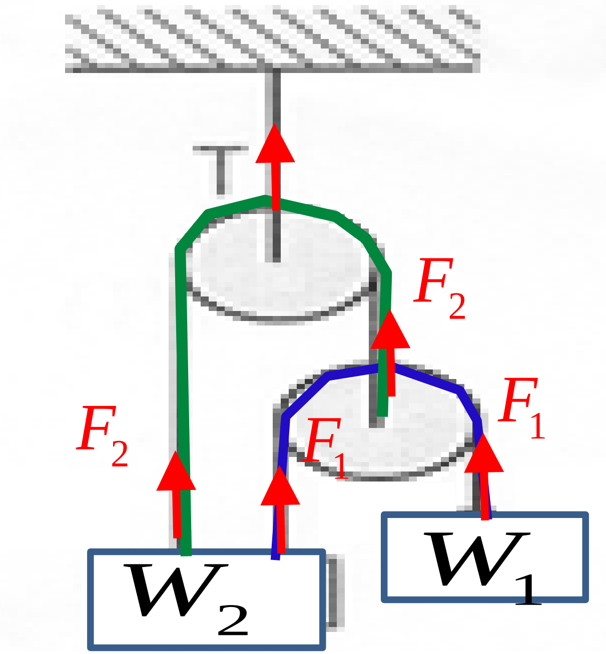
2.如圖(三)，已知W_1=2kgw，若滑輪重不計，且不考慮摩擦，試求

(1) W_2=

(2)最上端的張力 T =_________ 。



(三)



W_1:合力$=0$ $F_1 = W_1$

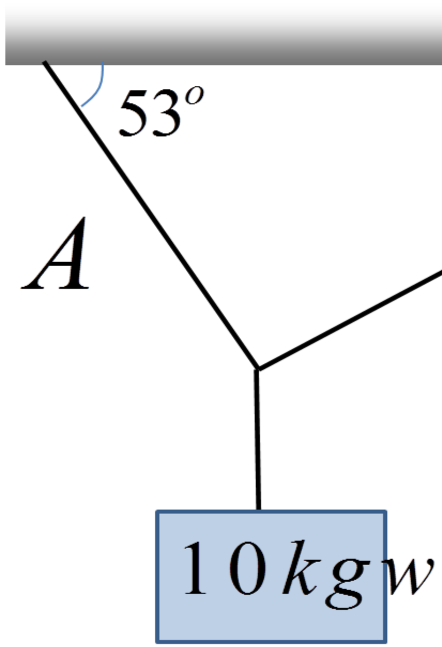
下滑輪:合力$=0$ $F_2 = 2F_1 = 2W_1$

W_2:合力$=0$ $F_1 + F_2 = W_2$

$\rightarrow W_1 + 2W_1 = W_2 \rightarrow 3W_1 = W_2$

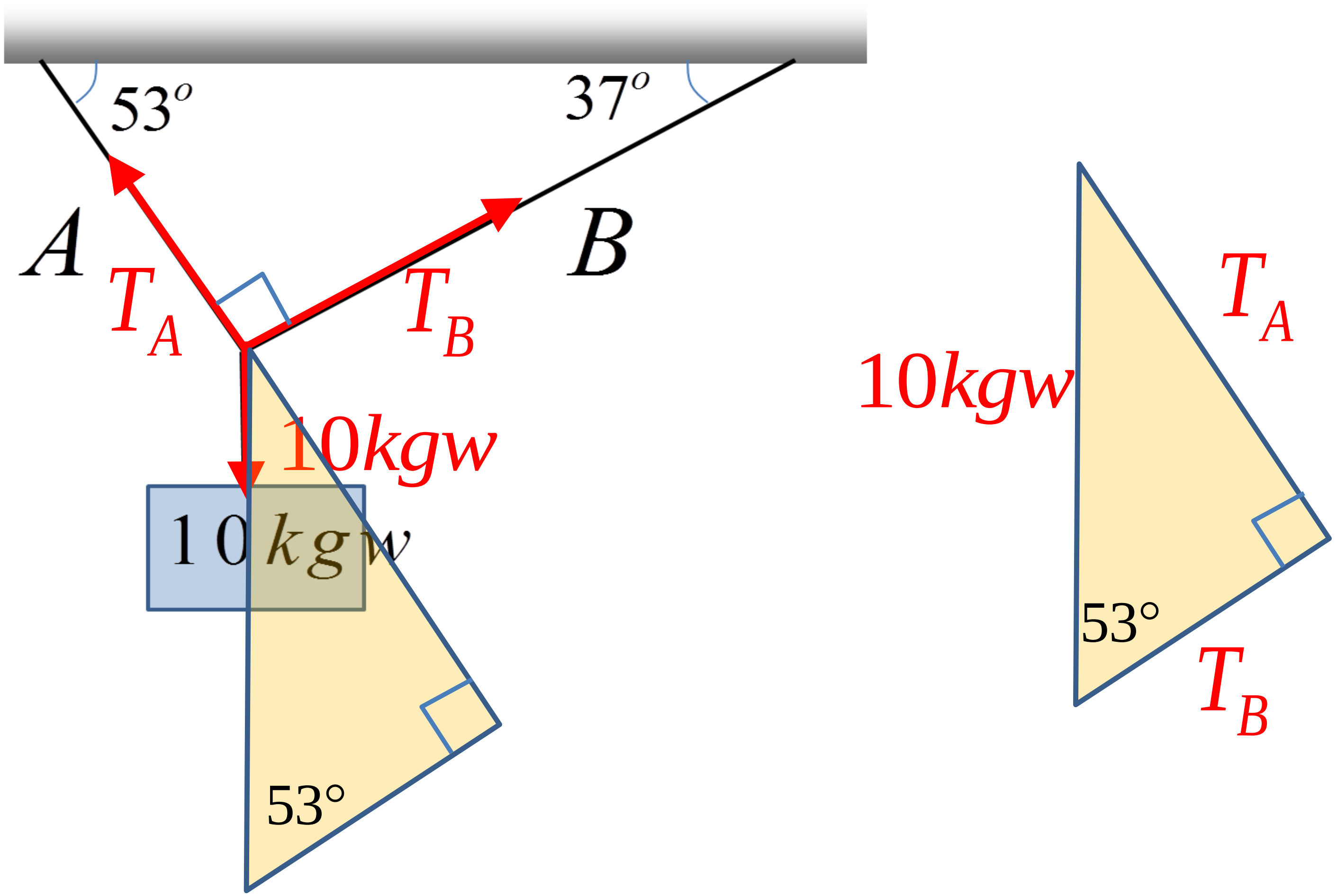
上滑輪:合力$=0$ $T = 2F_2 = 4W_1$

1.如圖，一重為10公斤重的物體掛在一繩上某點，恰可使之靜止不動，則繩作用於A、B兩點的張力各為多少公斤重？



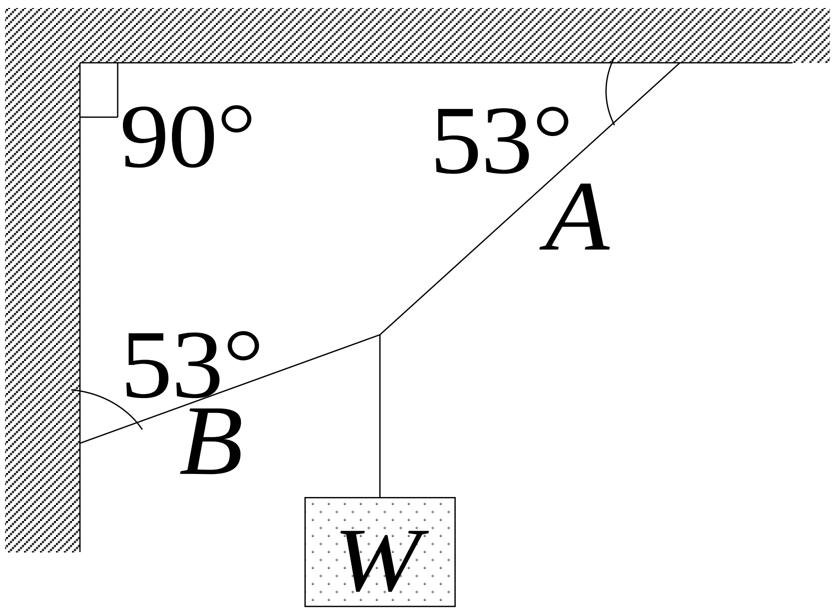
[解析]

幾何法



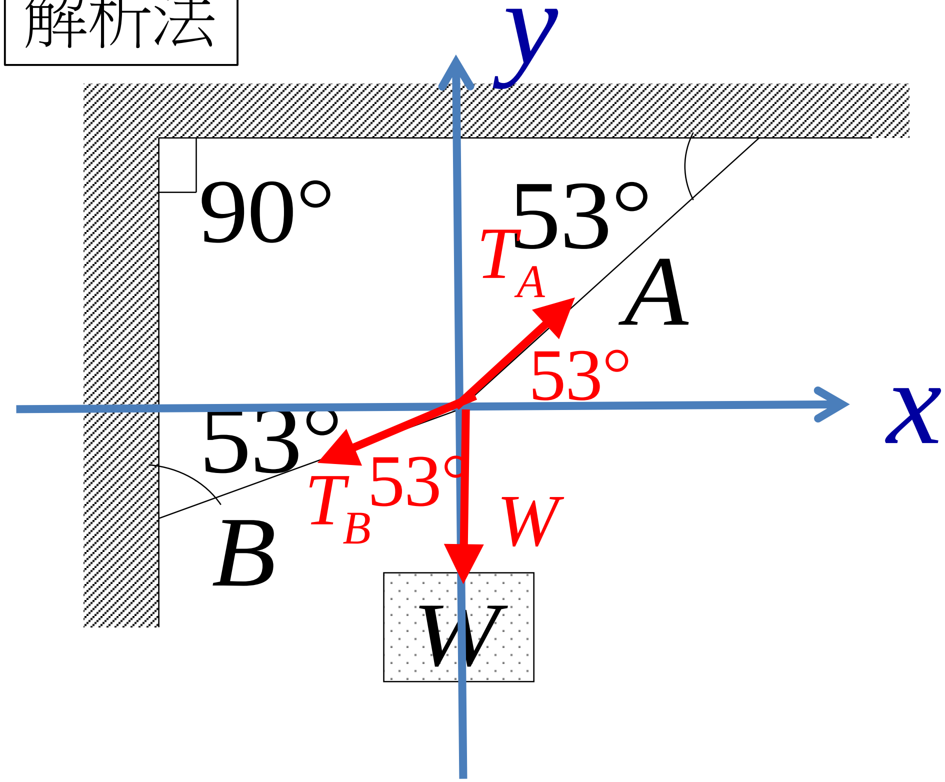
$$T_A = 10 \times \frac{4}{5} = 8 \quad T_B = 10 \times \frac{3}{5} = 6$$

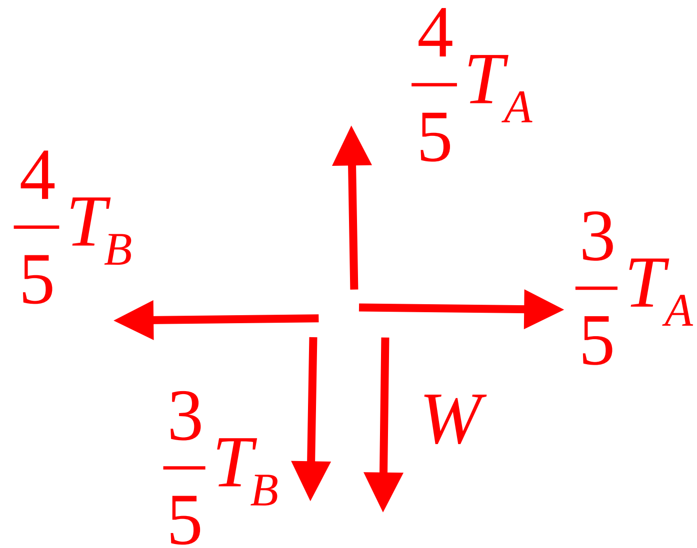
*2.*如圖之*A*、*B*兩繩吊一重物（重量為*W=350gw*）而成平衡。設繩上之張力大小各為T_A及T_B，則T_A＝____ T_B＝_____。



[解析]

解析法



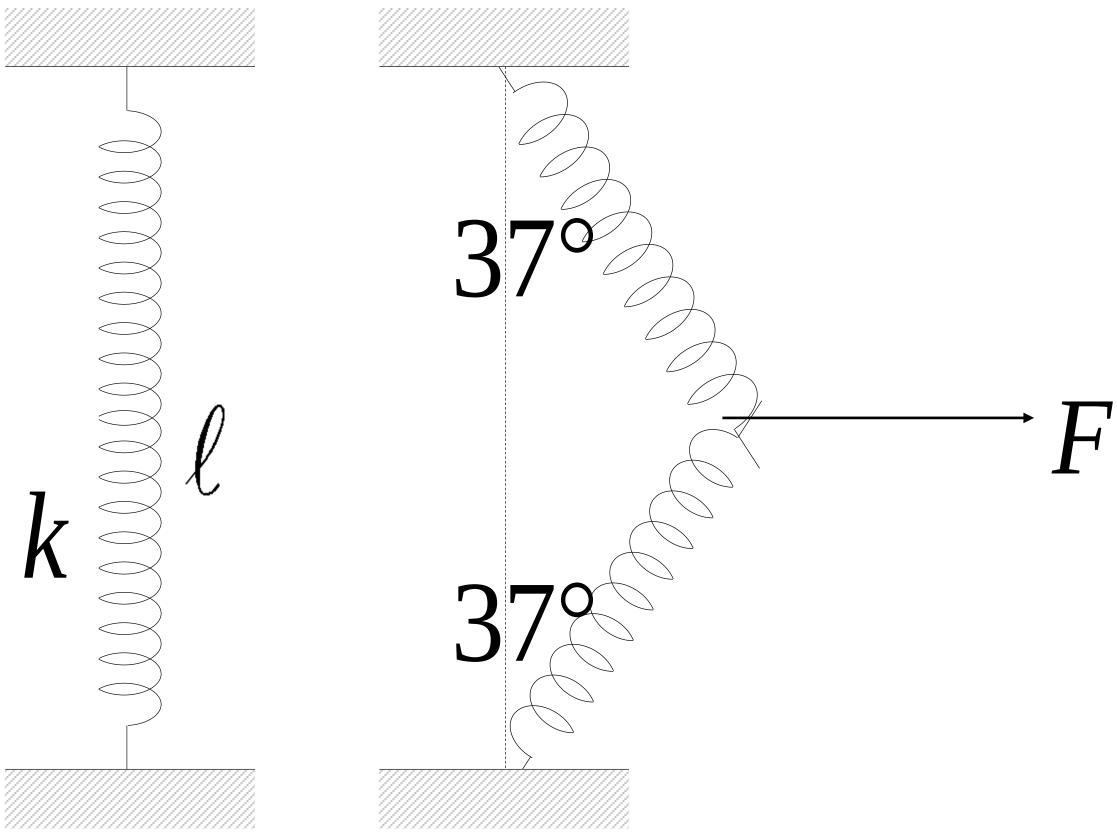


$$
合力=0\begin{cases} x: T_A\cos 53^\circ = T_B \sin 53^\circ \\ y: T_A \sin 53^\circ = T_B \cos 53^\circ + W \end{cases}
$$

$$\begin{cases} T_B = \dfrac{3}{4} T_A \\ T_A \times \dfrac{4}{5} = \dfrac{3}{4} T_A \times \dfrac{3}{5} + W \rightarrow \dfrac{7}{20} T_A = W \end{cases}$$

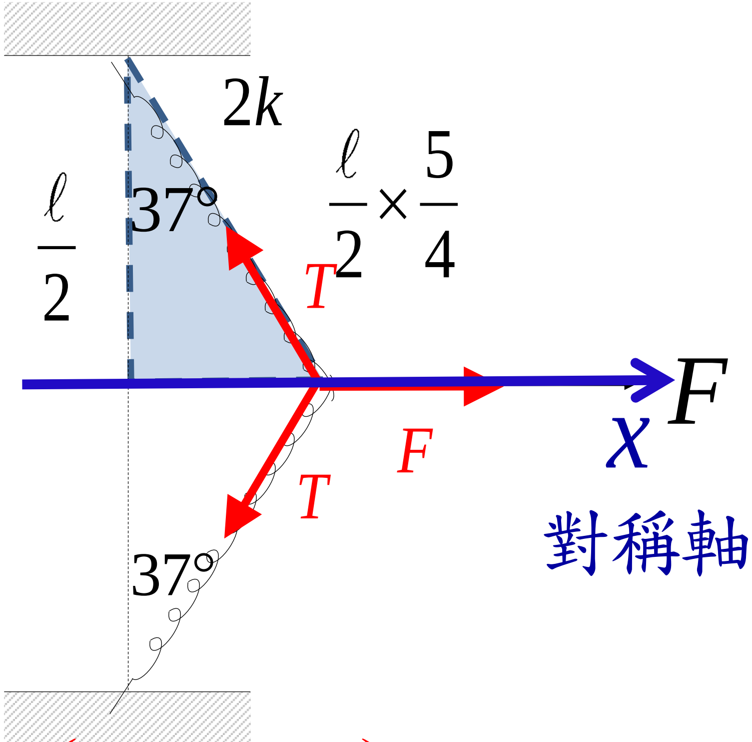
$$\therefore T_A = \frac{20}{7} W = 1000 \quad T_B = \frac{15}{7} W = 750$$

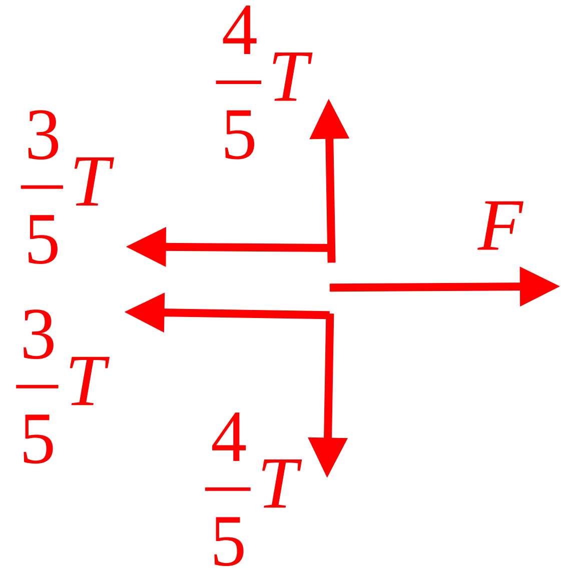
1.長 ℓ ，彈力常數*k*之輕彈簧，固定其上、下兩端，今於其中點處施一力*F*彈簧各段與鉛直線成*37°* 角，則*F*為？



[解析]

解析法



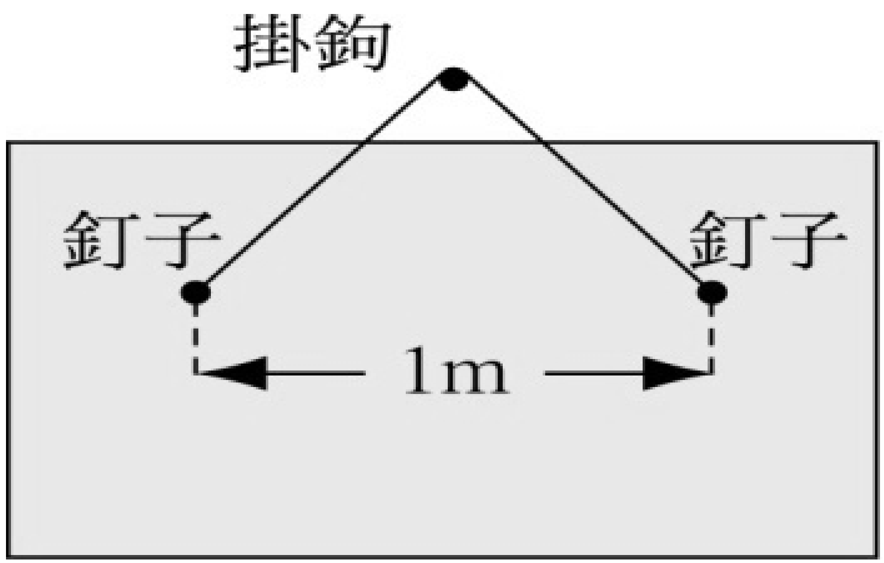


$$T = 2k \times \left(\frac{\ell}{2} \times \frac{5}{4} - \frac{\ell}{2} \right) = \frac{1}{4} k\ell$$

合力=0

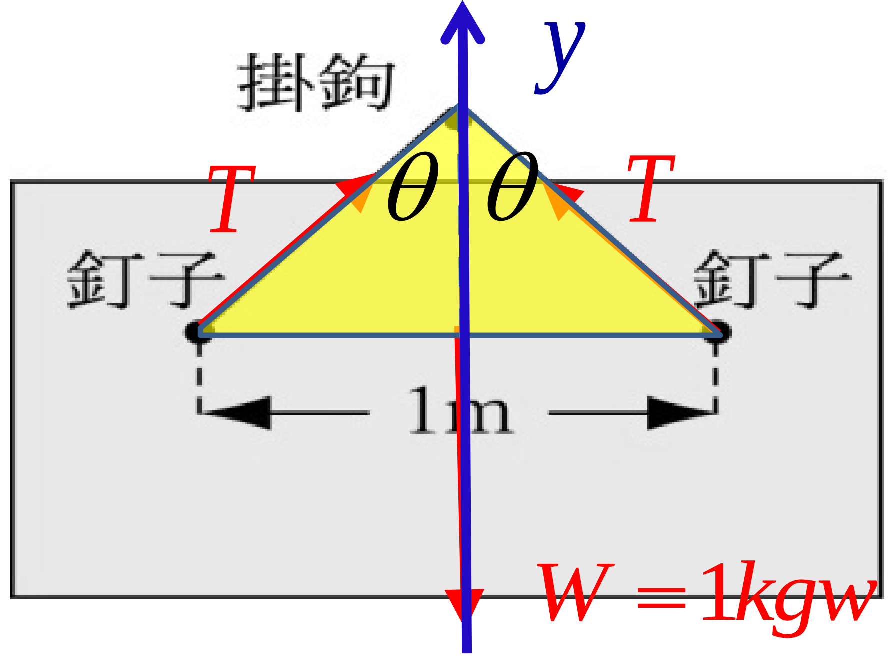
沿對稱軸 $x: F = 2T\sin 37° = 2 \times \frac{1}{4} k\ell \times \frac{3}{5} = \frac{3}{10} k\ell$

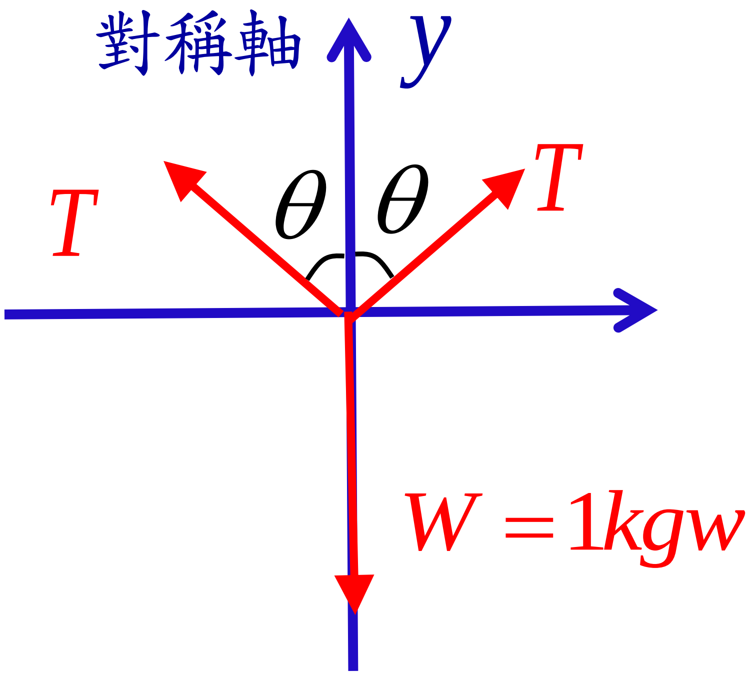
*2.*小軒要在客廳裏掛上一幅1 公斤重的畫(含畫框)， 畫框的背面有兩個相距1 公尺、位置固定的釘子。他將畫對稱的掛在牆壁的掛鉤上，掛繩最大可以承受1 公斤重的張力，掛好後整條細繩呈緊繃的狀態(如圖)。假設細繩可以承受的最大張力與繩長無關，則細繩最少需要幾公尺才不至於斷掉？

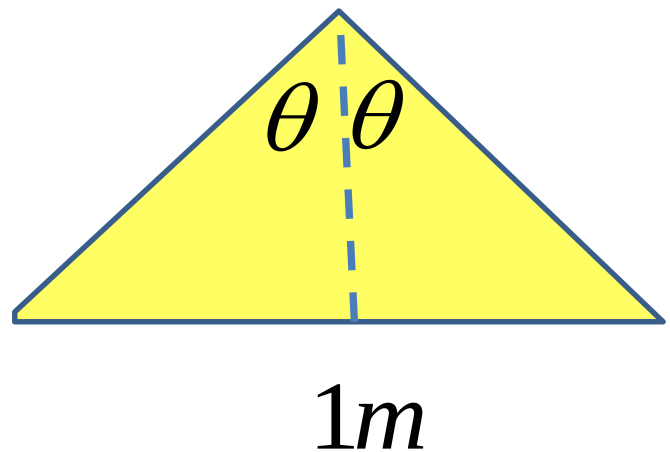


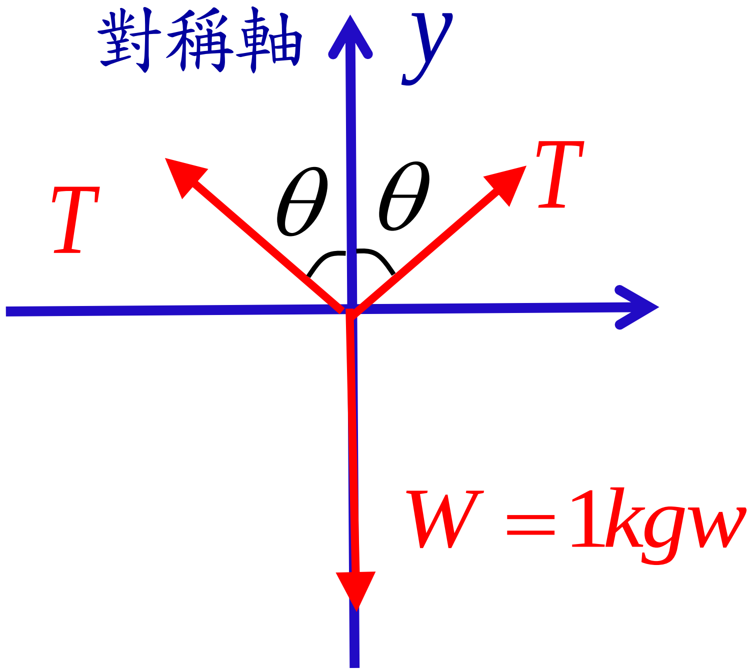
[解析]

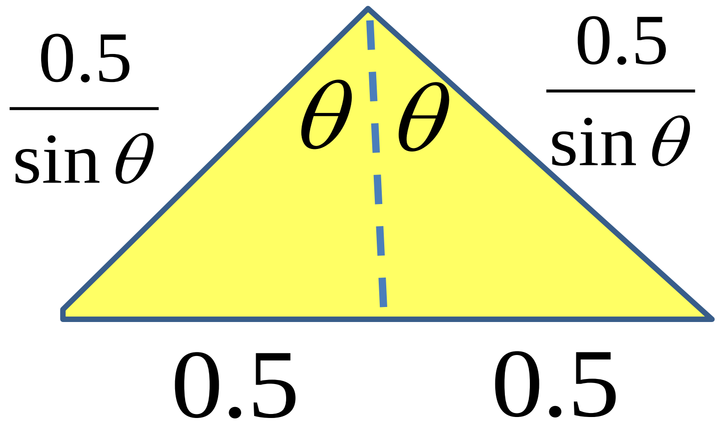
解析法







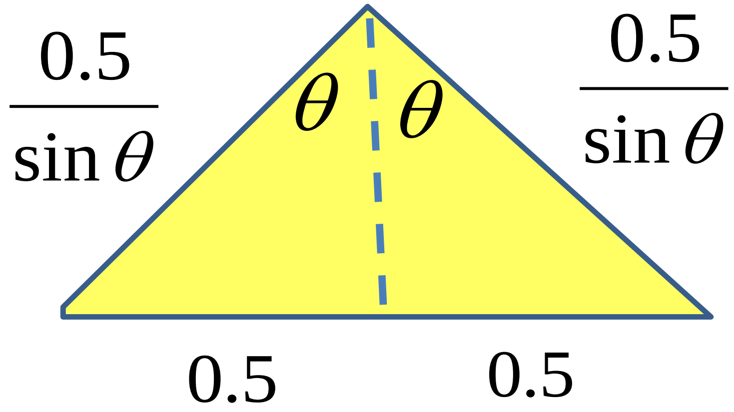




合力=0

沿對稱軸 $y: 2T\cos\theta = 1 \rightarrow T = \dfrac{1}{2\cos\theta} \leq 1$

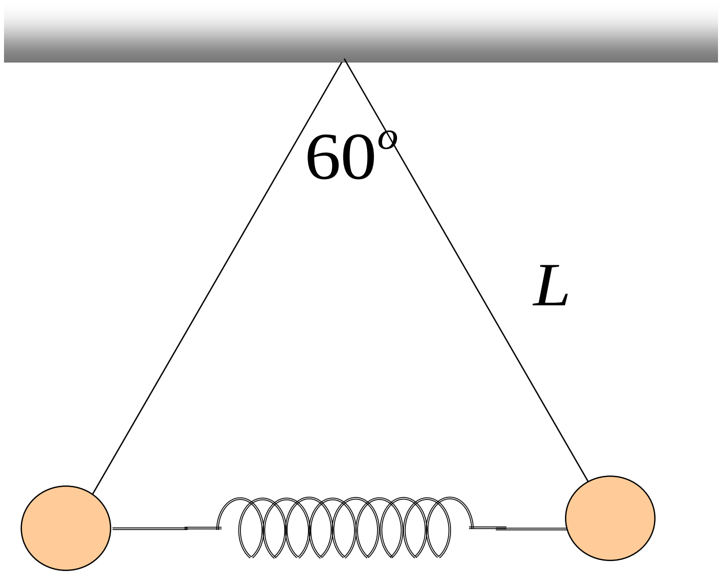
$$\cos\theta \geq \frac{1}{2} \rightarrow \theta \leq 60^\circ$$



當θ=60°時繩子最長

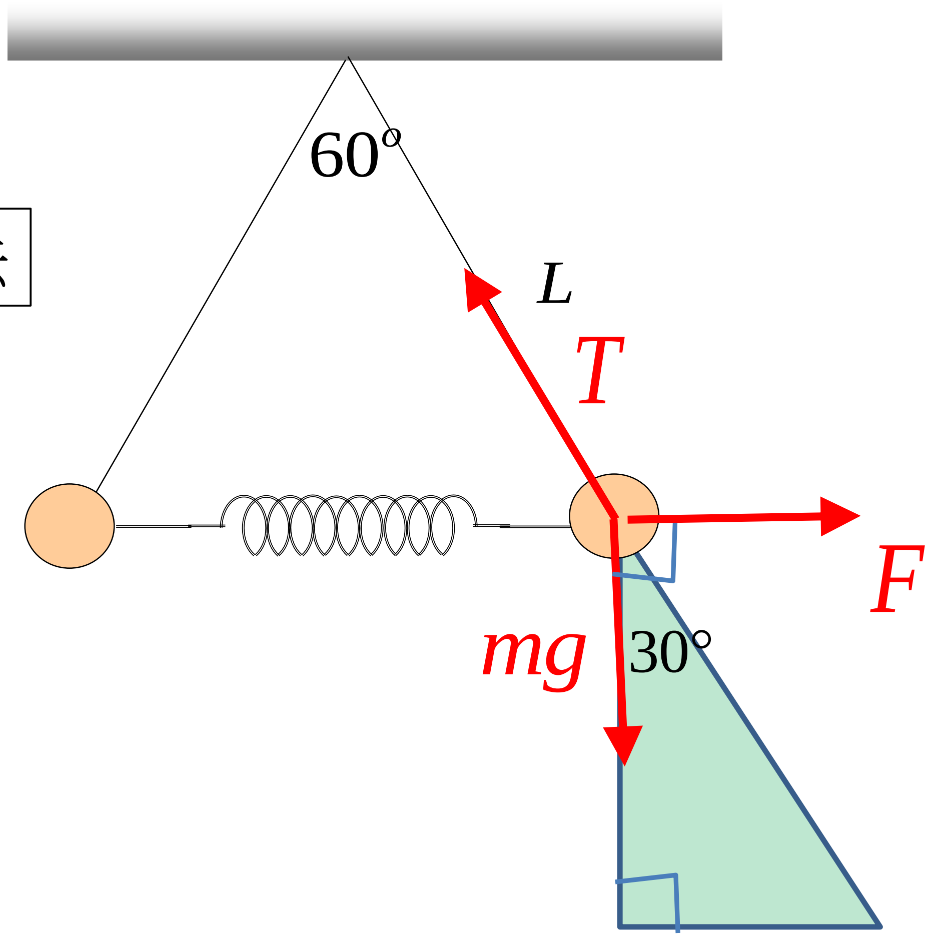
$$2\times\frac{0.5}{\sin 60^\circ}=\frac{2}{\sqrt{3}}$$

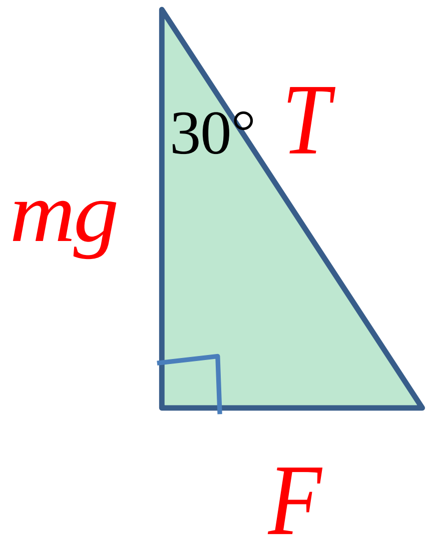
1. 質量均為*m*的*A*、*B*兩球，用兩根長度均為*L*的輕繩懸掛，兩球之間夾一力常數為*K*的 輕彈簧，平衡後夾 ，如圖所示，則輕彈簧被壓縮的長度是多少？（重力加速度g）



[解析]

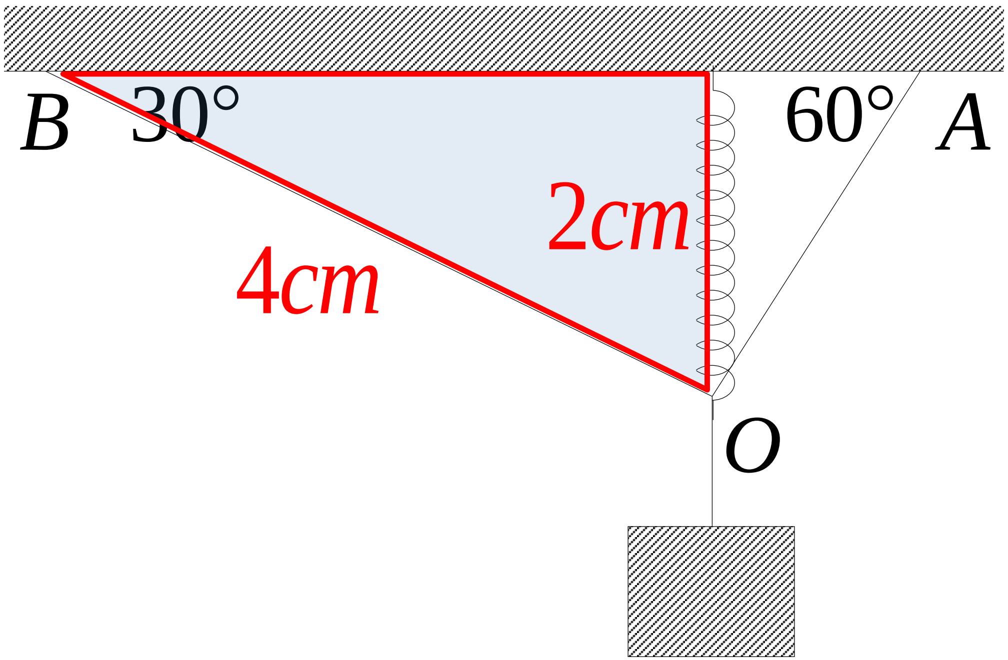
幾何法

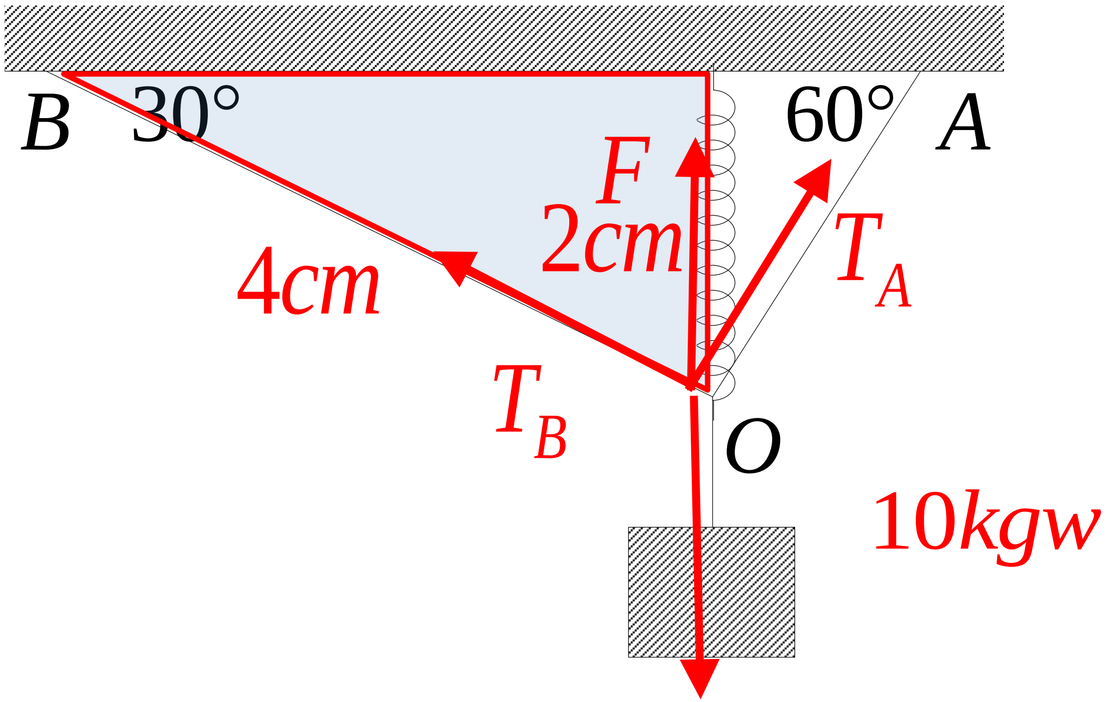




$$T = mg \times \frac{2}{\sqrt{3}} \quad F = mg \times \frac{1}{\sqrt{3}}$$

2. 物*10*公斤重，以細繩及彈簧吊起平衡如圖。設彈簧原長*1.5*厘米，彈力常數為*7840*牛頓／米，細繩較長者長度為*4*厘米，則較長細繩之張力為若干公斤重？

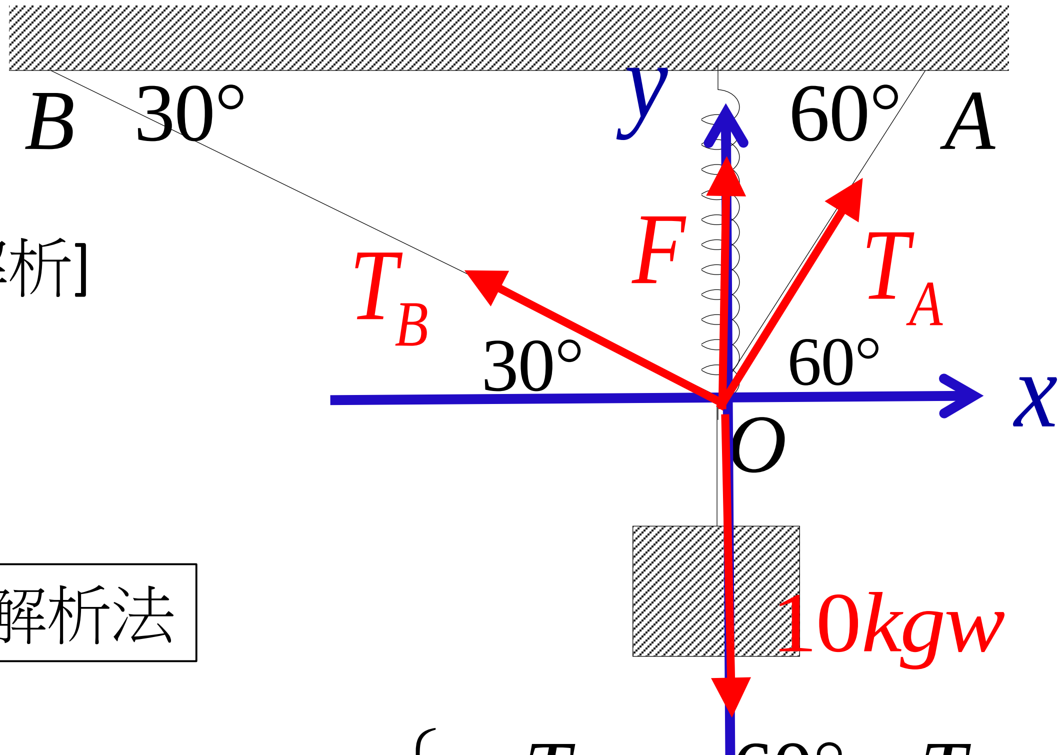


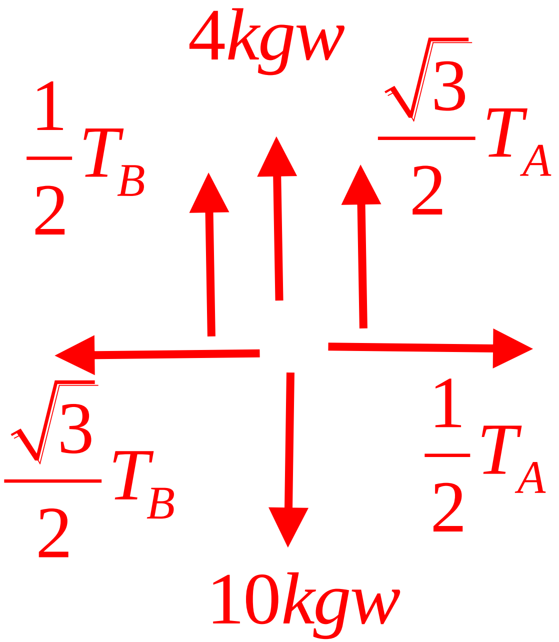


彈力常數 ***7840***牛頓／米= ***8***公斤重／厘米

$$F = kx = 8 \times (2 - 1.5) = 4[kgw]$$

[解析]





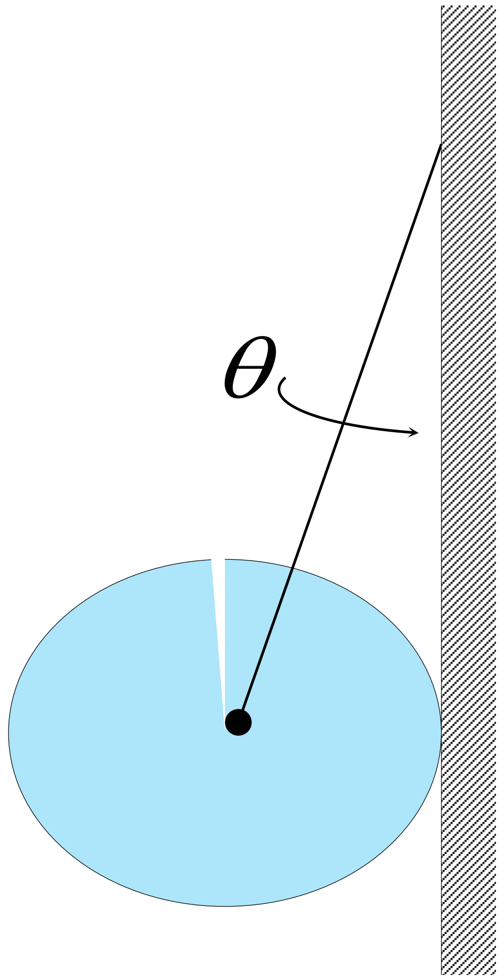
解析法

$$合力=0\begin{cases} x: T_A\cos 60^\circ = T_B\cos 30^\circ \\ y: T_A\sin 60^\circ + T_B sin30^\circ + F = 10 \end{cases}$$

$$\begin{cases} T_B = \dfrac{T_A}{\sqrt{3}} \\ \sqrt{3}T_A + T_B + 8 = 20 \end{cases} \qquad \therefore T_A = 3\sqrt{3} \quad T_B = 3$$

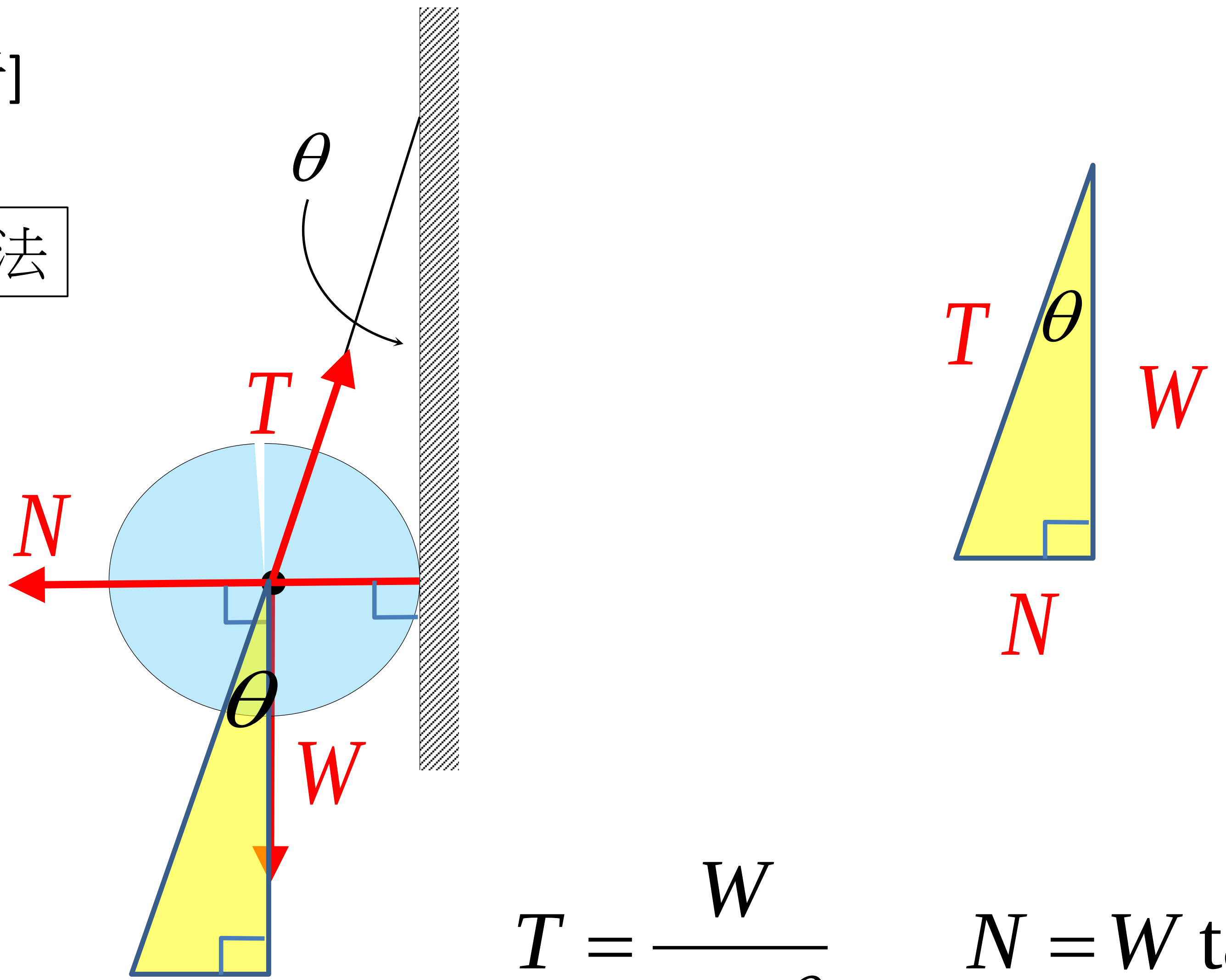
第101頁

*1.*球之重量為 *W* 懸掛於光滑垂直牆上，繩與牆夾角θ，如圖，則：(A) 繩張力？　(B) 牆作用於球正向力？(C) 若繩減短，則平衡時張力變大還是小？



[解析]

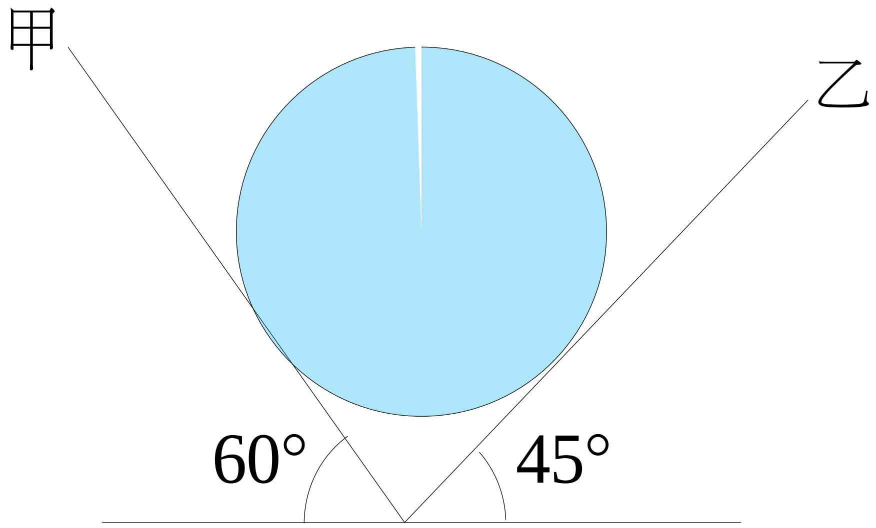
幾何法



$$T = \frac{W}{\cos\theta} \qquad N = W\tan\theta$$

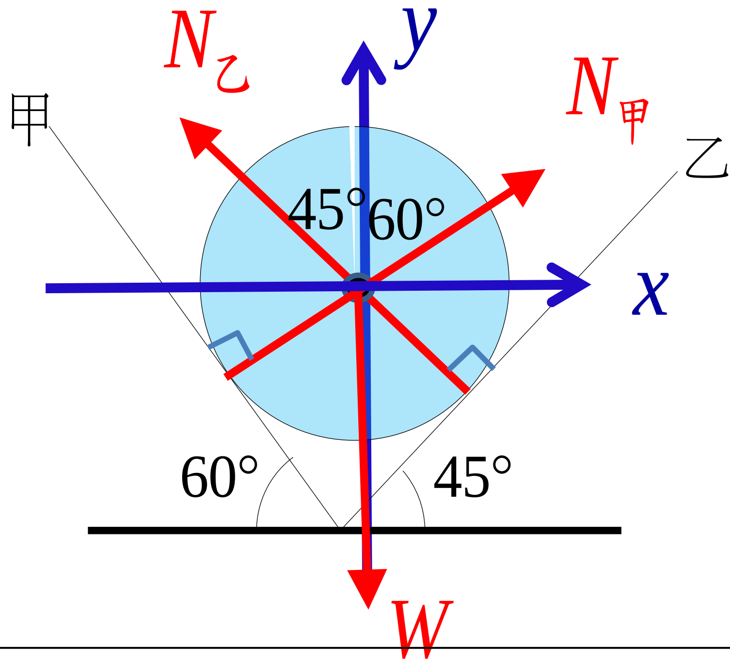
第101頁

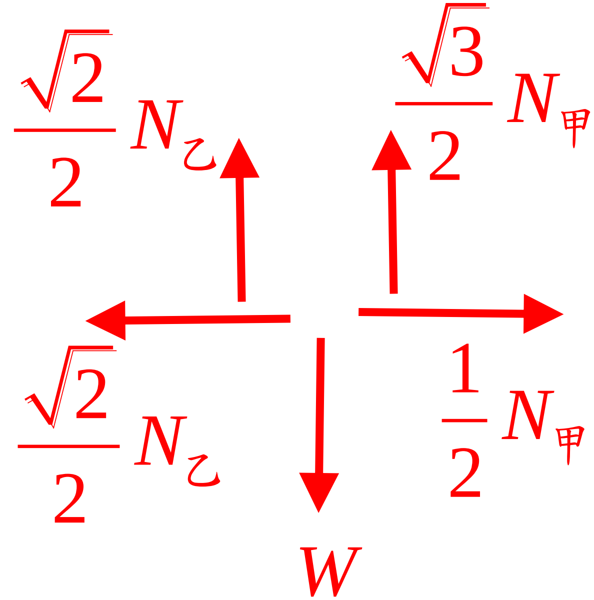
*2.*一重量為*W*之均勻圓球，架在底緣相靠之甲、乙兩光滑平板上，甲板與水平面成*60*°角，乙板與水平面成*45*°角(如圖)。設板與球間無摩擦力，則甲板施於球的作用力量值為？



[解析]

解析法



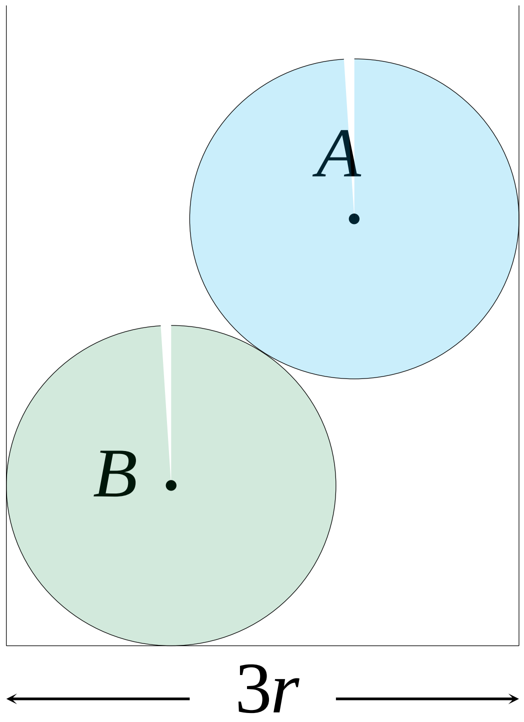


[關鍵]斜面正向力與鉛直線夾角=斜面傾斜角(斜面與水平夾角)

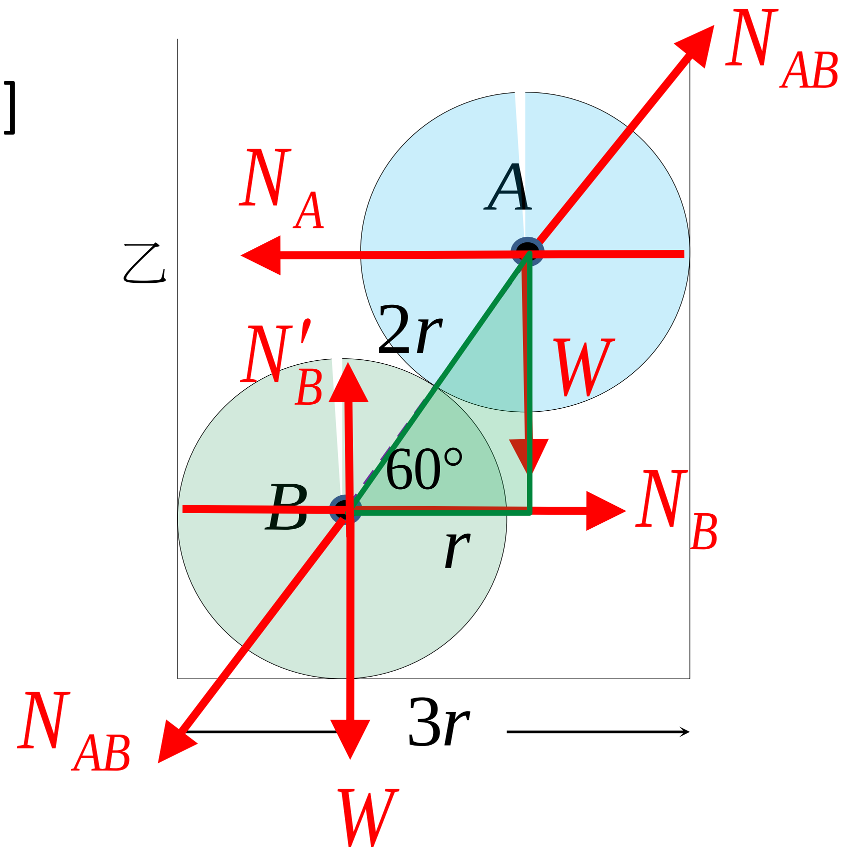
$$合力=0\begin{cases} x: N_甲 \sin 60^\circ = N_乙 \sin 45^\circ \\ y: N_甲 \cos 60^\circ + N_乙 \cos 45^\circ = W \end{cases}$$

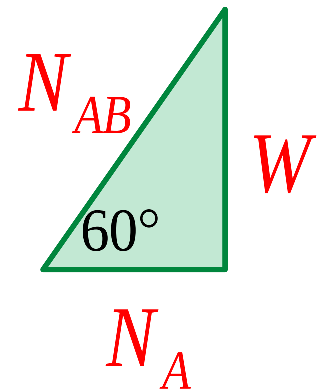
$$\begin{cases} \sqrt{3}N_甲 = \sqrt{2}N_乙 \\ N_甲 + \sqrt{2}N_乙 = 2W \end{cases} \therefore N_甲 = \frac{2W}{1+\sqrt{3}} \quad N_乙 = \frac{2\sqrt{3}W}{\sqrt{2}+\sqrt{6}}$$

1. 如圖所示*A*，*B*兩球均重W置於底邊*3r* (*r*為*A*，*B*兩球之半徑)之容器中，則：(A)容器底作用於*B*球之力為？
(B) *A*，*B*間之作用力？
(C) 左右容器壁對球之力各為？



[解析]

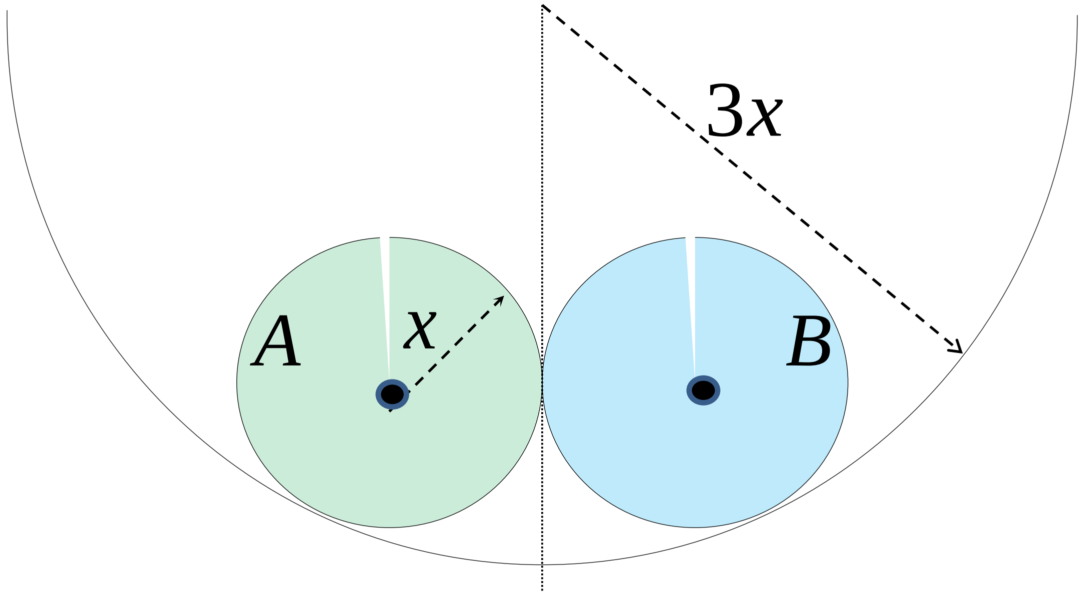




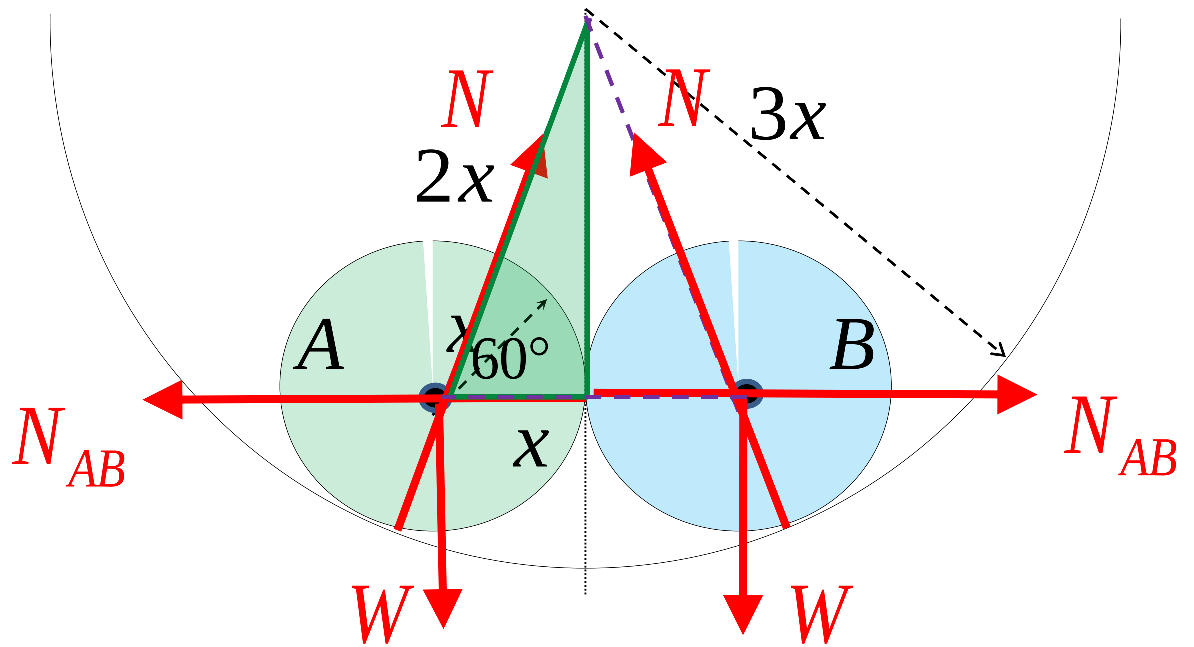
解析法 $A+B$ ： $\text{合力}=0\begin{cases} x: N_A = N_B \\ y: N'_B = 2W \end{cases}$

幾何法 A ： $N_{AB}=\dfrac{2}{\sqrt{3}}W \quad N_A=\dfrac{1}{\sqrt{3}}W$

2.重量*W*，半徑*x*之兩球*A*、*B*置於半徑*3x*之半球形碗內(如圖)，則：
(A) *B*對*A*之作用力為？
(B) 碗壁對*B*之作用力為？。

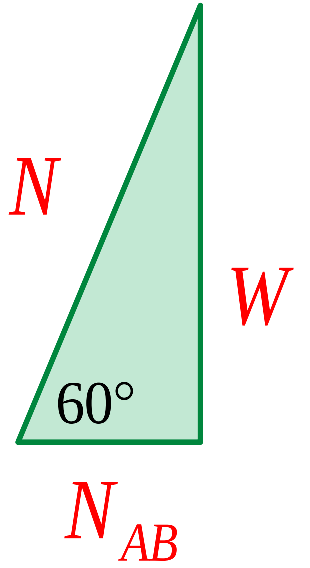


[解析]



幾何法

$$A：N=\frac{2}{\sqrt{3}}W \quad N_{AB}=\frac{1}{\sqrt{3}}W$$

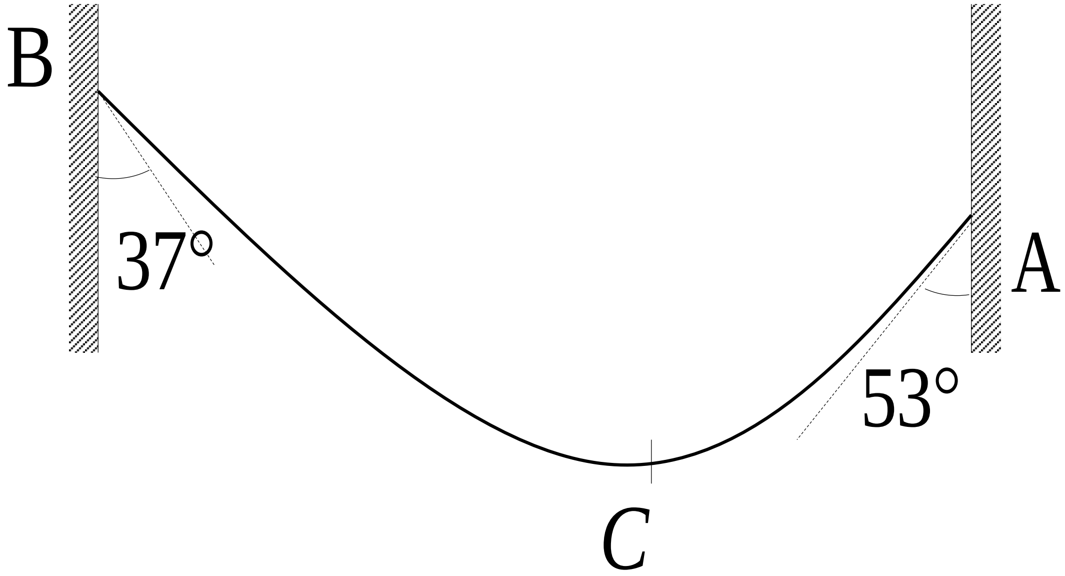


第103頁

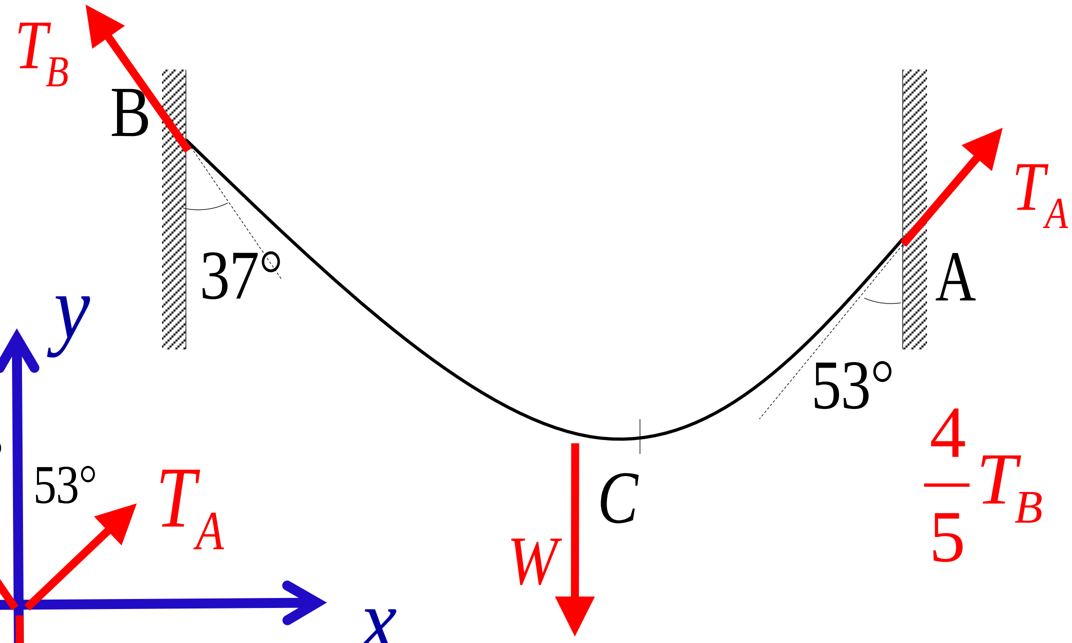
如圖，一柔軟均勻質量鐵鍊，懸吊於二牆之間，鍊重W，A點切線與牆夾角53°，B點切線與牆夾角37°，C為最低點，則

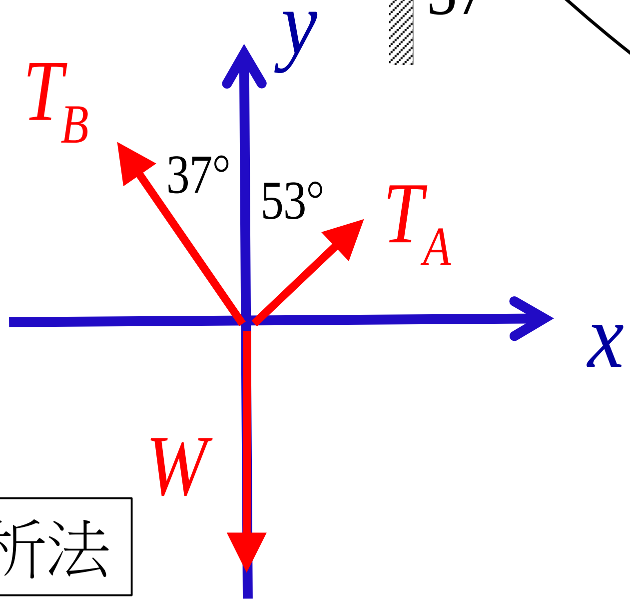
（A）A點之張力為？（B）B點之張力為？（C）C點之張力為？

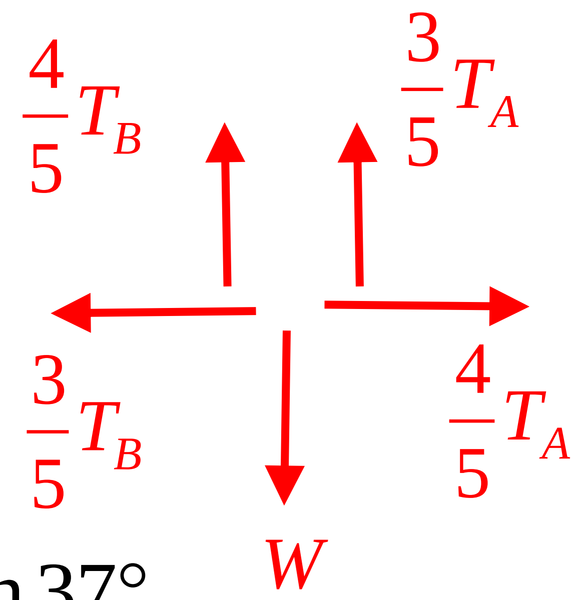
（D）AC段重與BC段重比？



[解析]



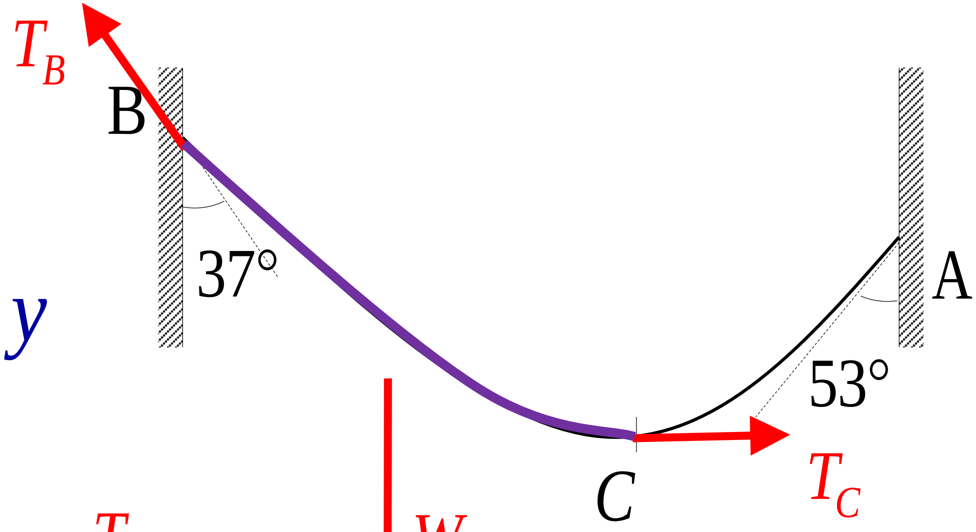


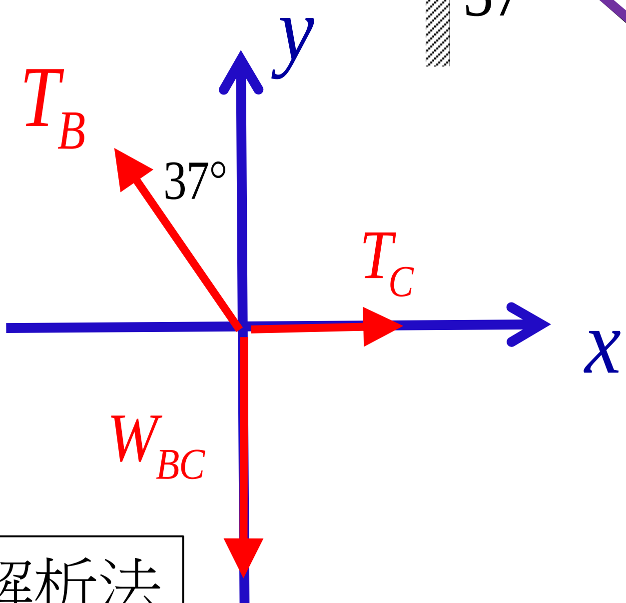


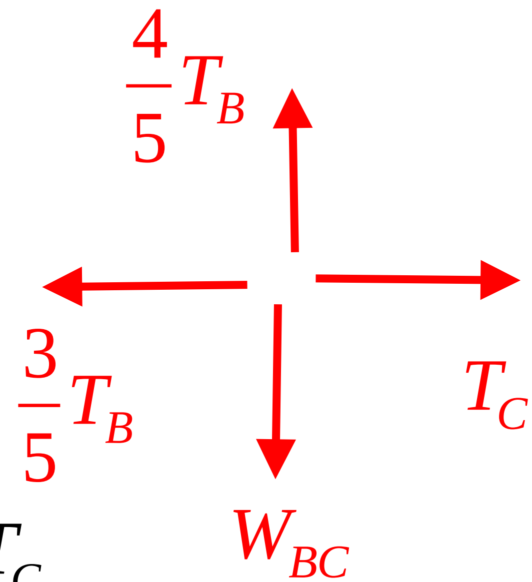
解析法

$$\text{全繩 合力}=0\begin{cases} x: T_A\sin 53° = T_B\sin 37° \\ y: T_A\cos 53° + T_B\cos 37° = W \end{cases}$$

$$\therefore T_A = \frac{3}{5}W \quad T_B = \frac{4}{5}W$$





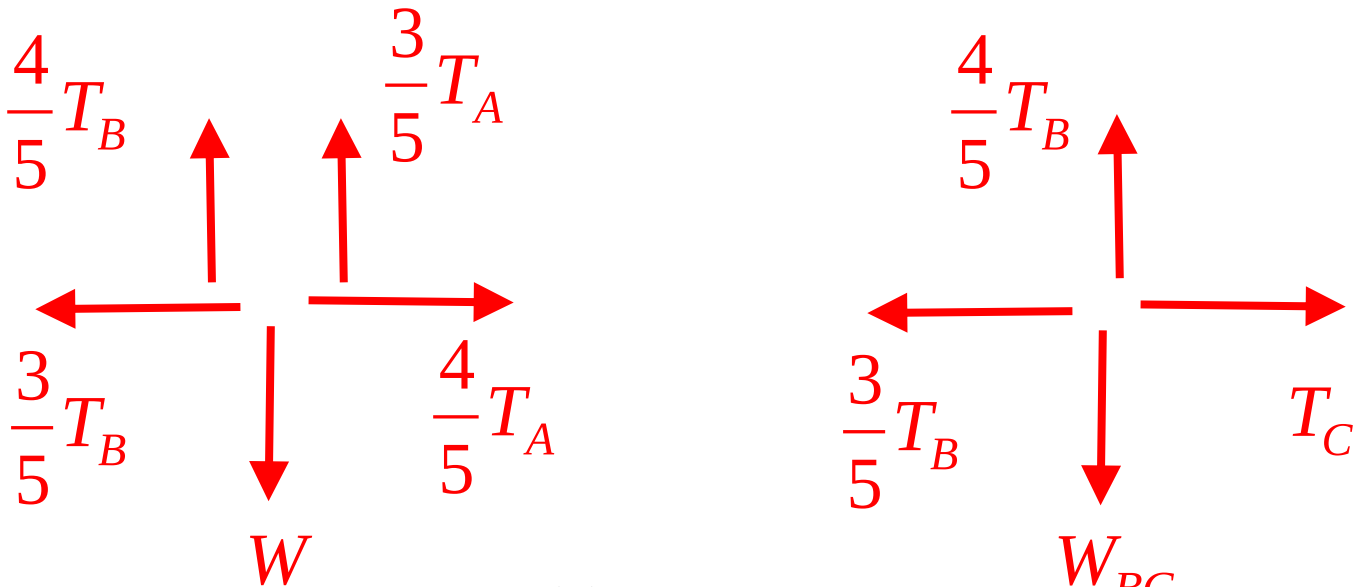


解析法

$$\text{BC繩　合力}=0\begin{cases} x: T_B \sin 37° = T_C \\ y: T_B \cos 37° = W_{BC} \end{cases}$$

$$\therefore T_C = \frac{12}{25}W \quad W_{BC} = \frac{16}{25}W$$

[討論]



(1) 繩張力水平分量大小相等(= 最低點繩張力):

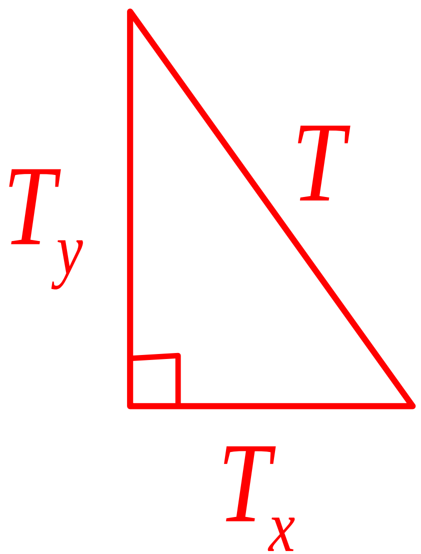
$$\frac{4}{5}T_A=\frac{3}{5}T_B=T_C$$

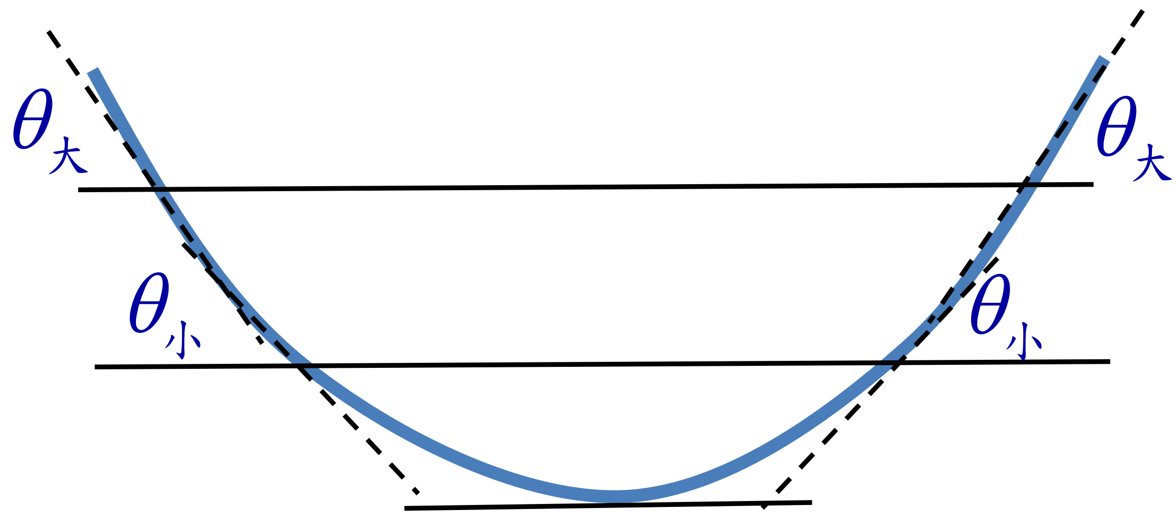
(2) A繩張力鉛直分量大小 = AC繩重量: $\frac{4}{5}T_A=W_{AC}$

B繩張力鉛直分量大小 = BC繩重量: $\frac{3}{5}T_B=W_{BC}$

繩上各點張力鉛直分量大小越高越大

[結論]



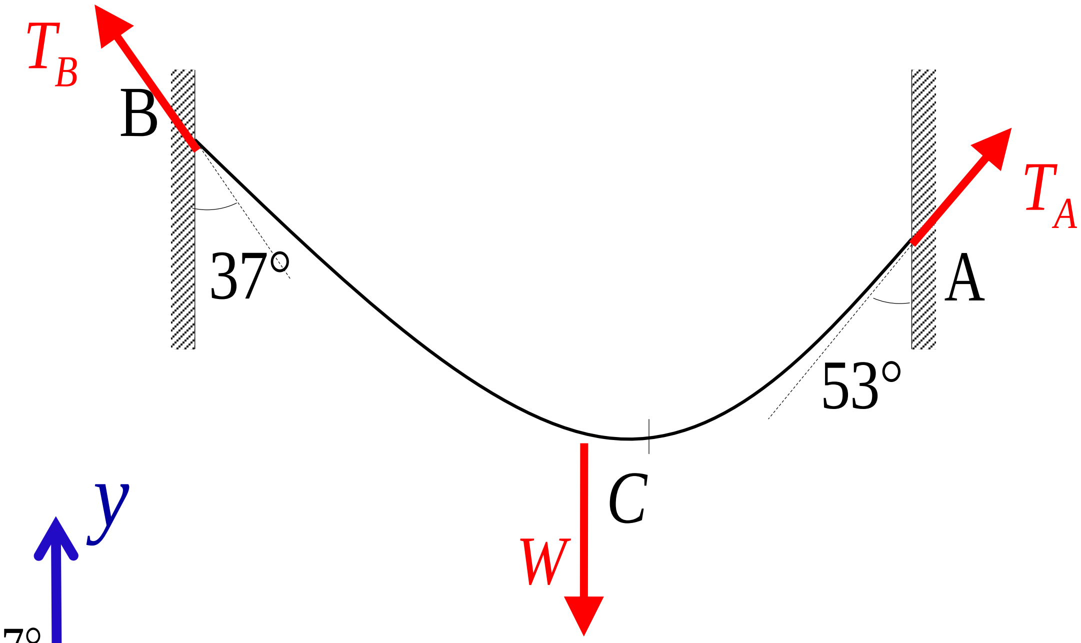


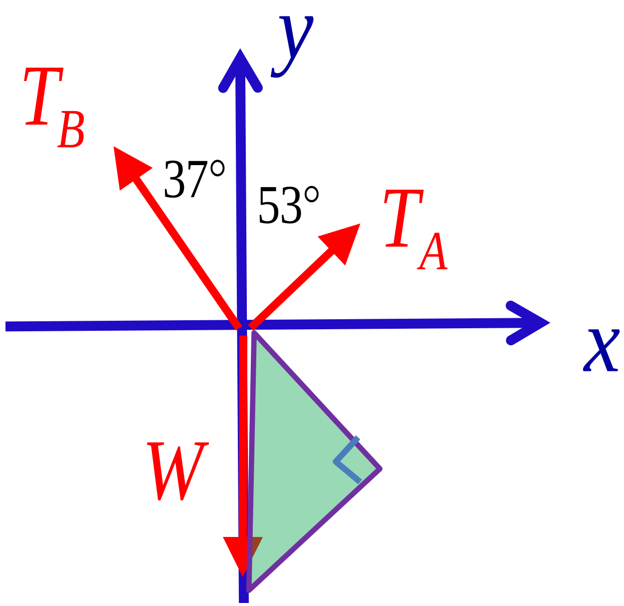
繩上各點張力水平分量大小相同
繩上各點張力鉛直分量大小越高越大

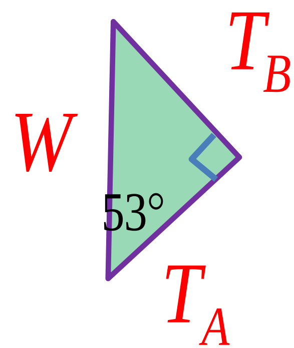
各點張力越高越大

繩子切線斜率越高越大

[另解]



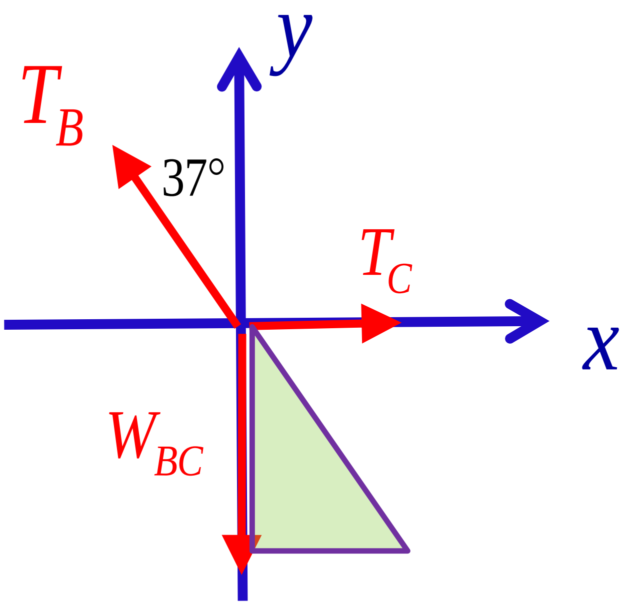


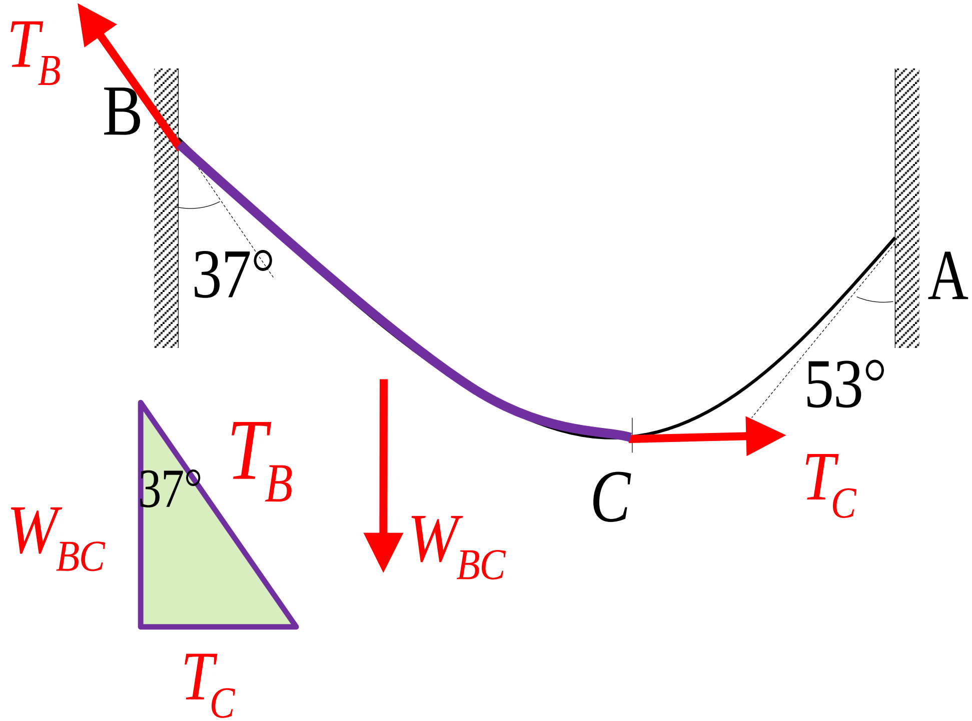


$\boxed{\text{幾何法}}$

全繩：$T_A = \dfrac{3}{5}W \quad T_B = \dfrac{4}{5}W$

[另解]





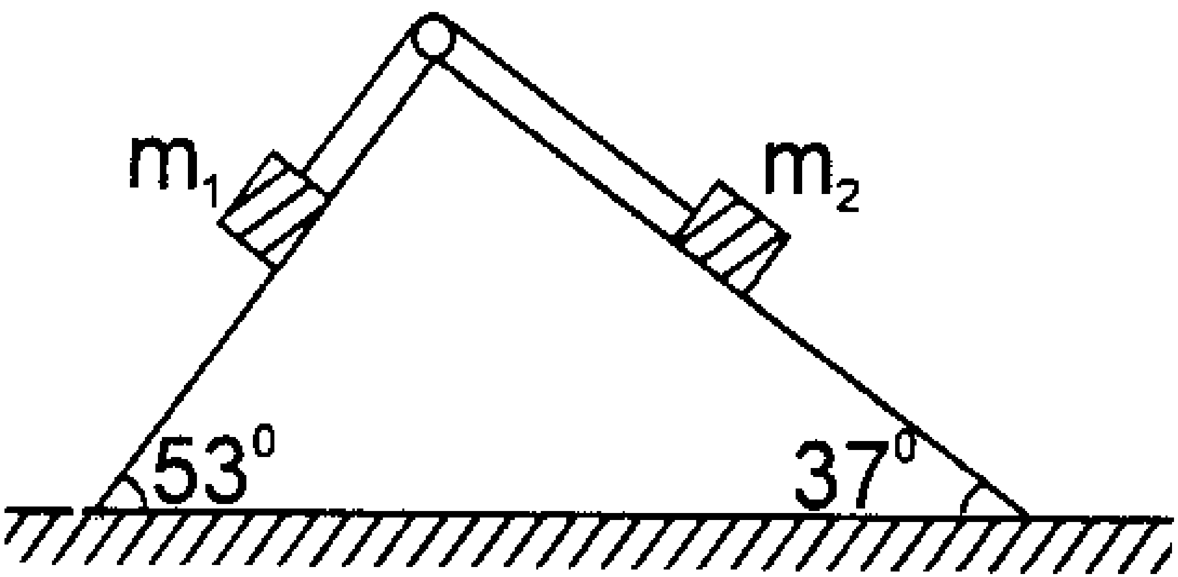
幾何法

$$\text{BC繩}：T_C = \frac{3}{5}T_B = \frac{3}{5}\times\frac{4}{5}W = \frac{12}{25}W$$

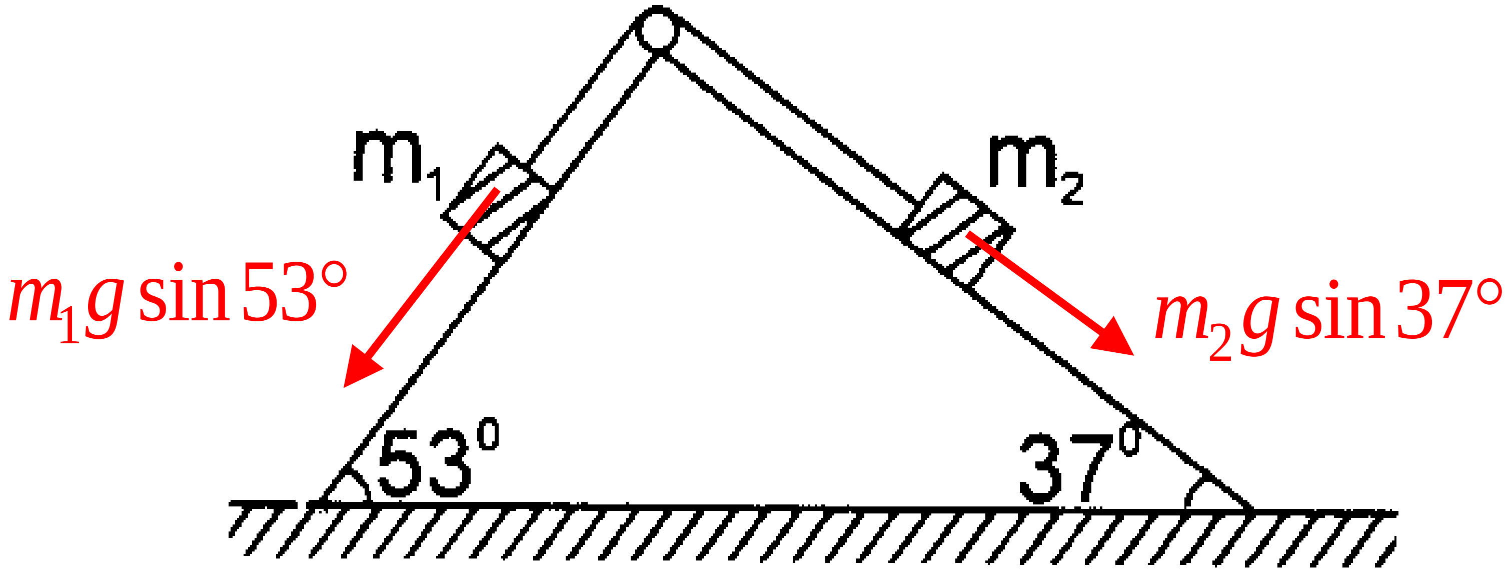
$$W_{BC} = \frac{4}{5}T_B = \frac{4}{5}\times\frac{4}{5}W = \frac{16}{25}W$$

$$\therefore W_{AC} = W - W_{BC} = W - \frac{16}{25}W = \frac{9}{25}W$$

1.考慮如圖所示系統，設各接觸面均無摩擦，若此質量系統保持平衡，則m_1及m_2的比值如何?



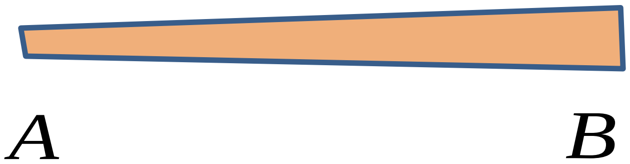
[解析]



$$m_1 g\sin 53° = m_2 g\sin 37°$$

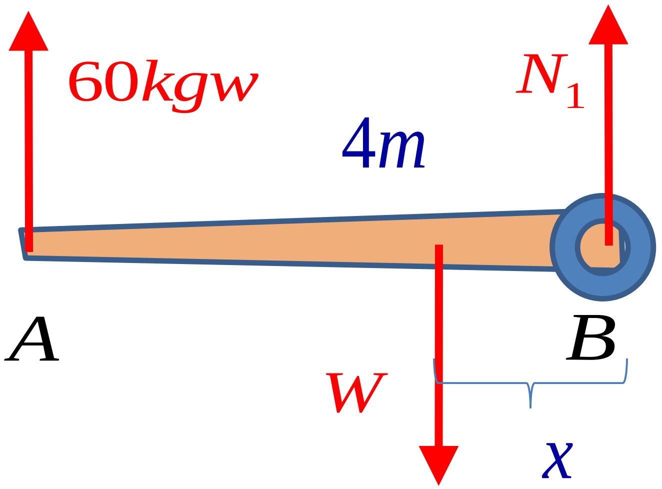
$$\therefore m_1 : m_2 = \sin 37° : \sin 53° = 3:4$$

*1.*一非均勻的木棒*AB*，長4*m*，欲將*A*端提起，至少需力60*kgw*，欲將*B*端提起，則最少需力80*kgw*，則木棒重多少？



[解析]　令木棒重量W，重心距B端x　重心視為重力作用點

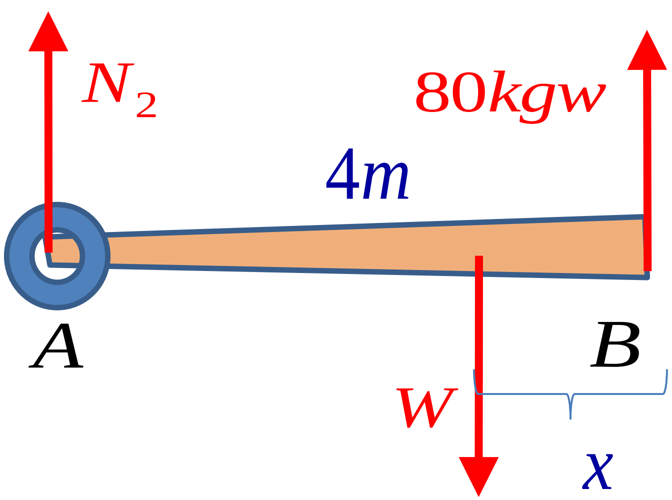
當恰拉起時，木棒與桌面的正向力恰通過轉軸



以B端為轉軸　恰拉起時

合力矩$=0$

$$60\times 4=W\times x$$



以A端為轉軸　恰拉起時

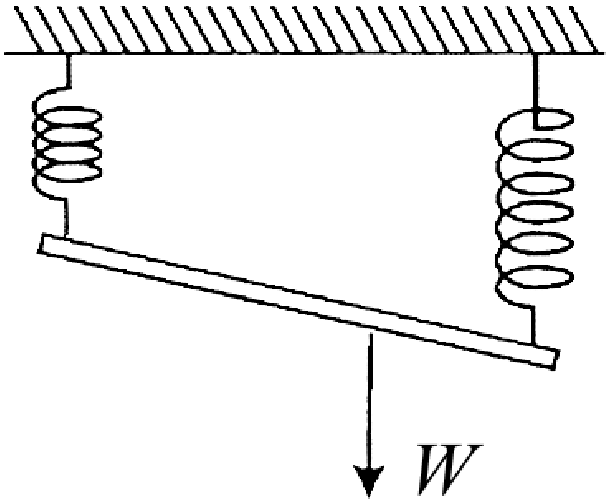
合力矩$=0$

$$80\times 4=W\times (4-x)$$

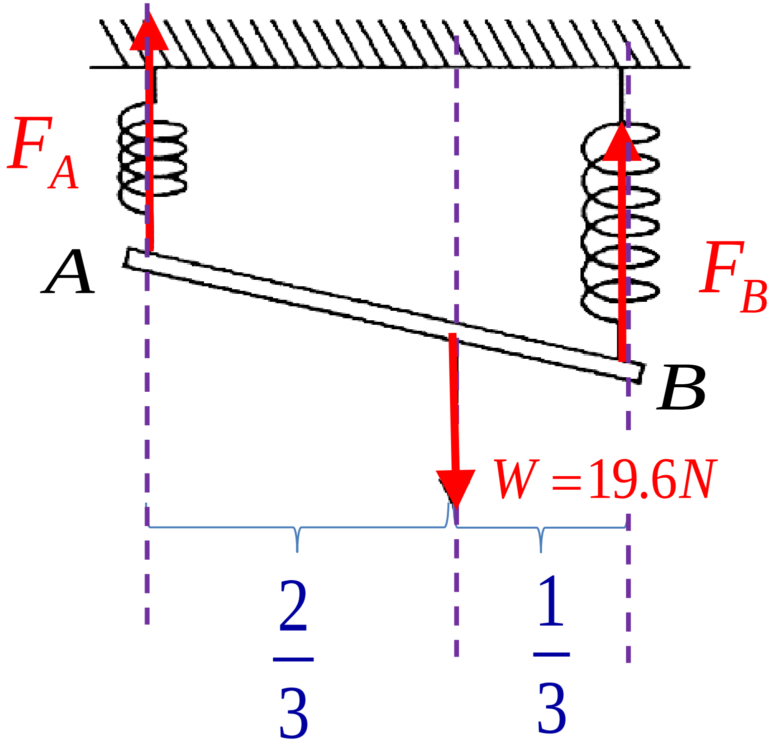
$$\therefore W=60+80=140$$

$$\therefore x=\frac{12}{7}$$

2. 重19.6*N*的木棒，用*k*=400*N*/*m*的兩相同彈簧吊起，棒的重心在一端 $\frac{1}{3}$ 長度處，則兩彈簧的伸長量約相差相多少厘米？



[解析]



以B端為轉軸 合力矩$=0$

$$19.6\times\frac{1}{3}=F_A\times 1\rightarrow F_A=\frac{19.6}{3}$$

以A端為轉軸 合力矩$=0$

$$19.6\times\frac{2}{3}=F_B\times 1\rightarrow F_B=\frac{39.2}{3}$$

$$F_A=kx_A\rightarrow x_A=\frac{F_A}{k}\quad F_B=kx_B\rightarrow x_B=\frac{F_B}{k}$$

$$\therefore x_B-x_A=\frac{F_B-F_A}{k}=\frac{\frac{39.2-19.6}{3}}{400}=\frac{49}{3000}[m]$$

第106頁

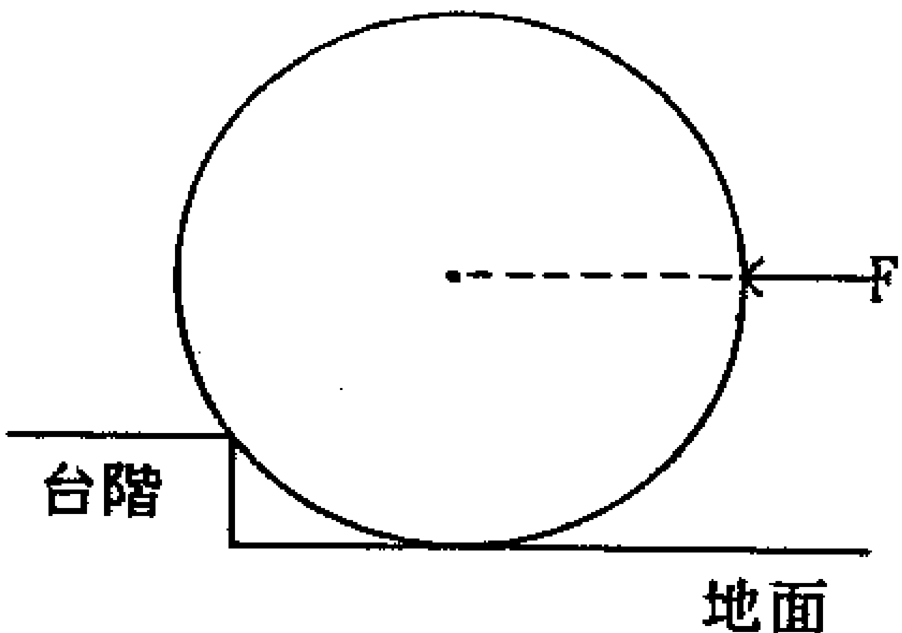
圖中，施一力*F*，欲將一半徑*50cm*，重*15kgw*之圓筒推置高出地面*20cm*之台階上，設階緣與筒不互相滑動，當筒恰離開地面時，試問：

（1）若F為水平力且通過圓筒軸心，F至少為___kgw，台階對筒所施之力為____ kgw。

（2）承（1），若F=10kgw，則地面與圓筒的作用力為___，台階對筒所施之力為____ kgw。

（3）若F為水平力，F至少為___ kgw，台階對筒所施之力為____ kgw。

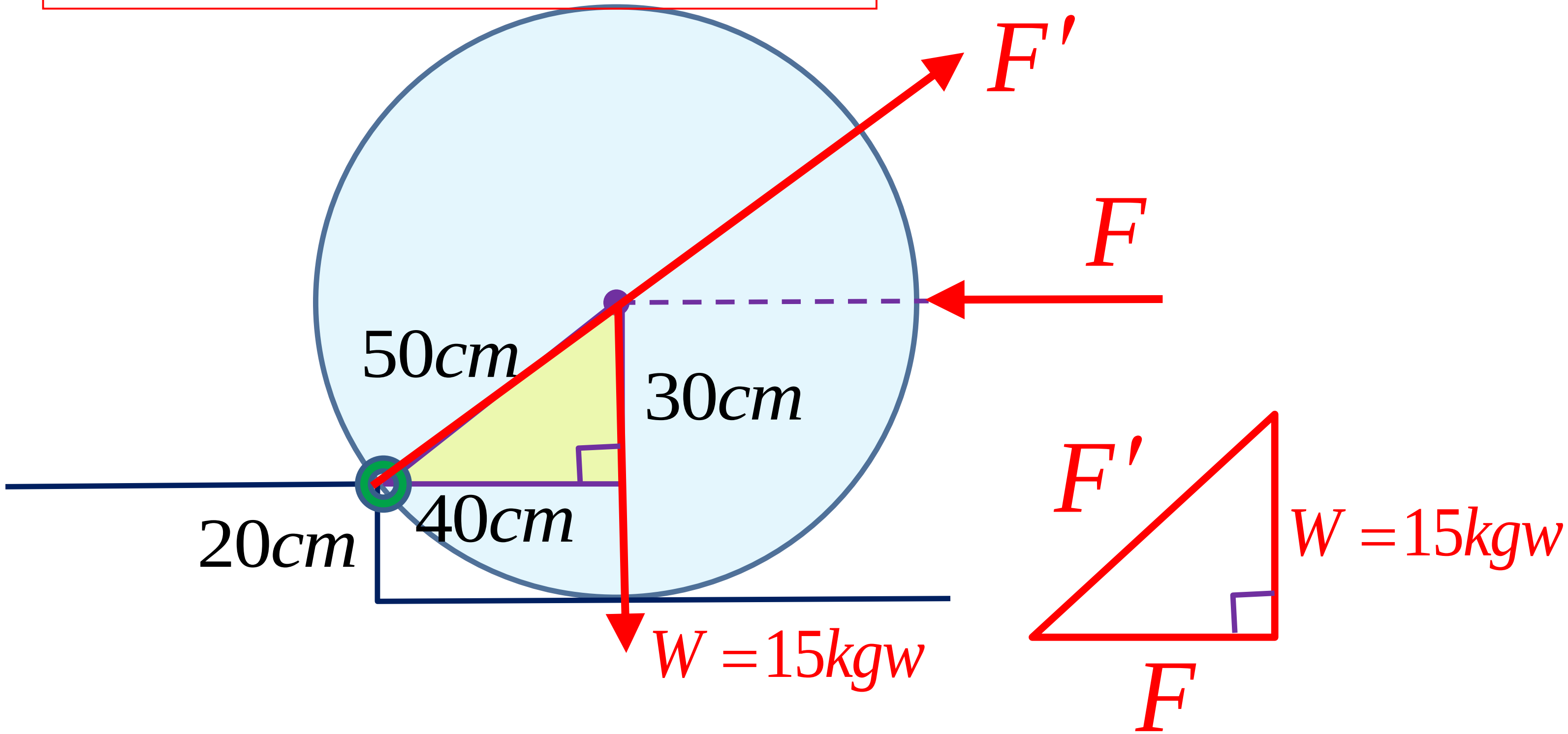
（4）若F非水平力， F至少為____ kgw，台階對筒所施之力為____ kgw



不平行三力平衡必交於一點

[解析]

（1）



當正要推上時，圓筒與地面的正向力恰為零

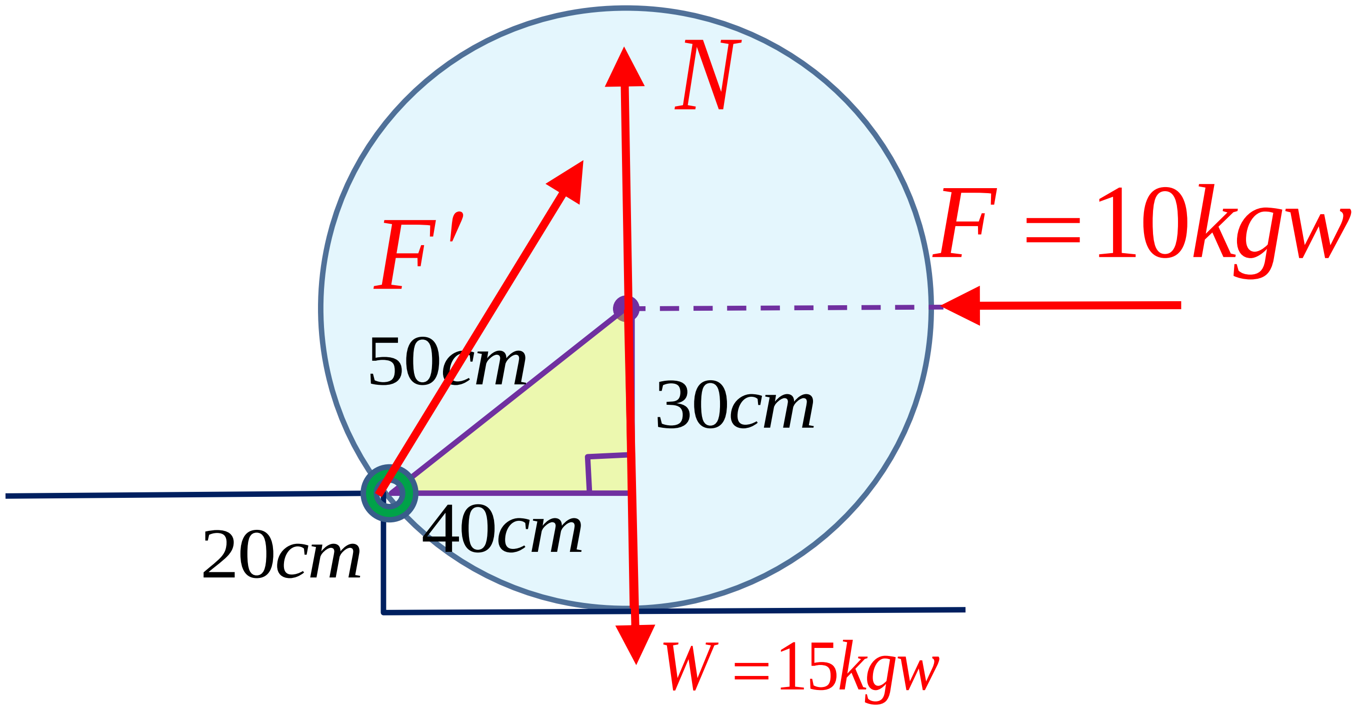
以階緣為轉軸 恰拉起時

合力矩$=0$ $15\times40=F\times30$ $\therefore F=20$

$\therefore$階緣作用力$F'=\sqrt{F^2+W^2}=\sqrt{20^2+15^2}=25$

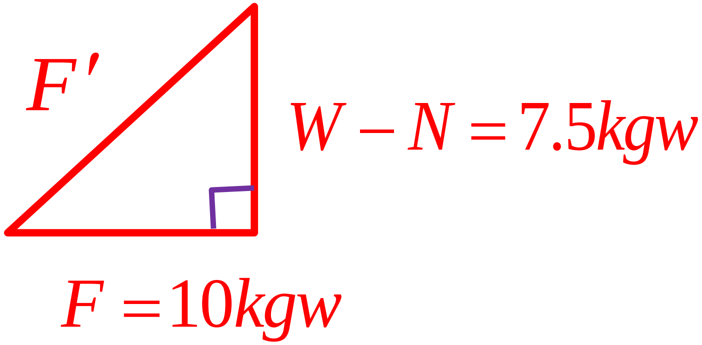
[解析]

（2）



令圓筒與地面的正向力N

以階緣為轉軸　合力矩$=0$

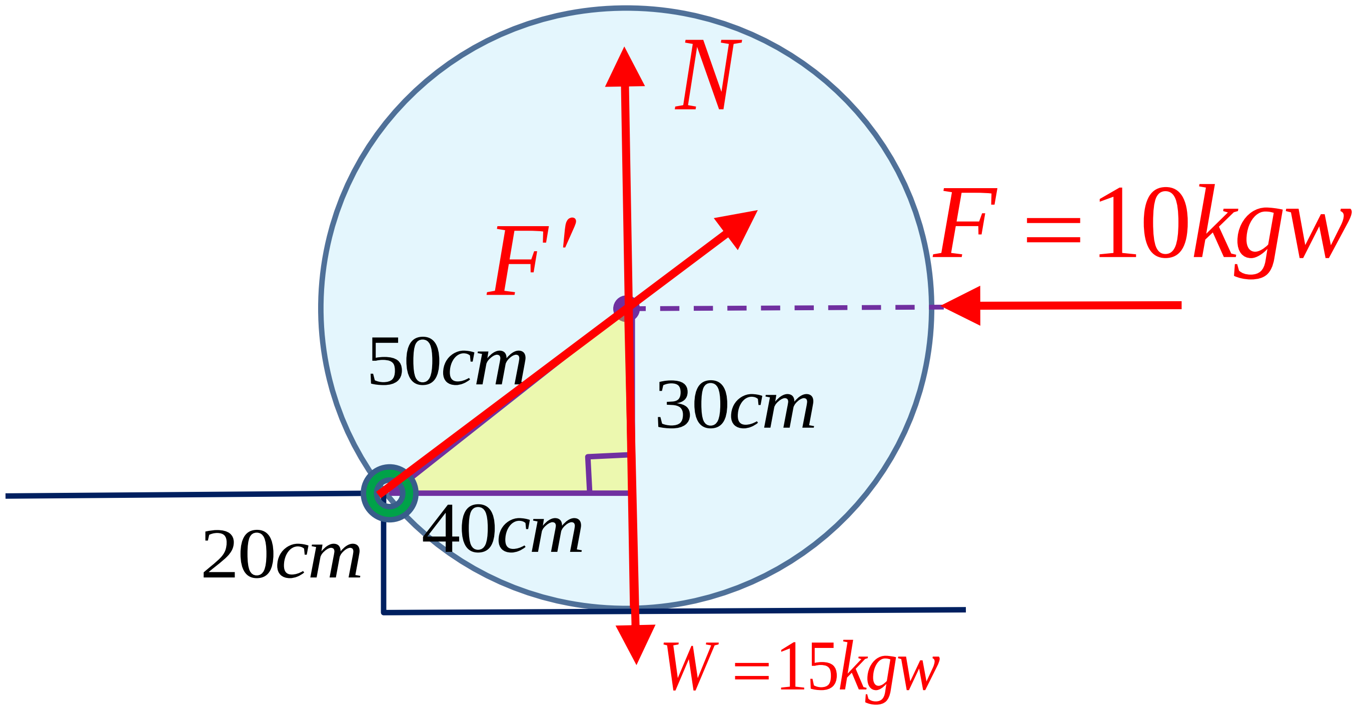


$(15-N)\times 40=10\times 30 \quad \therefore N=7.5$

$\therefore$階緣作用力$F'=\sqrt{F^2+(W-N)^2}=\sqrt{10^2+(15-7.5)^2}=12.5$

[解析]

（2）



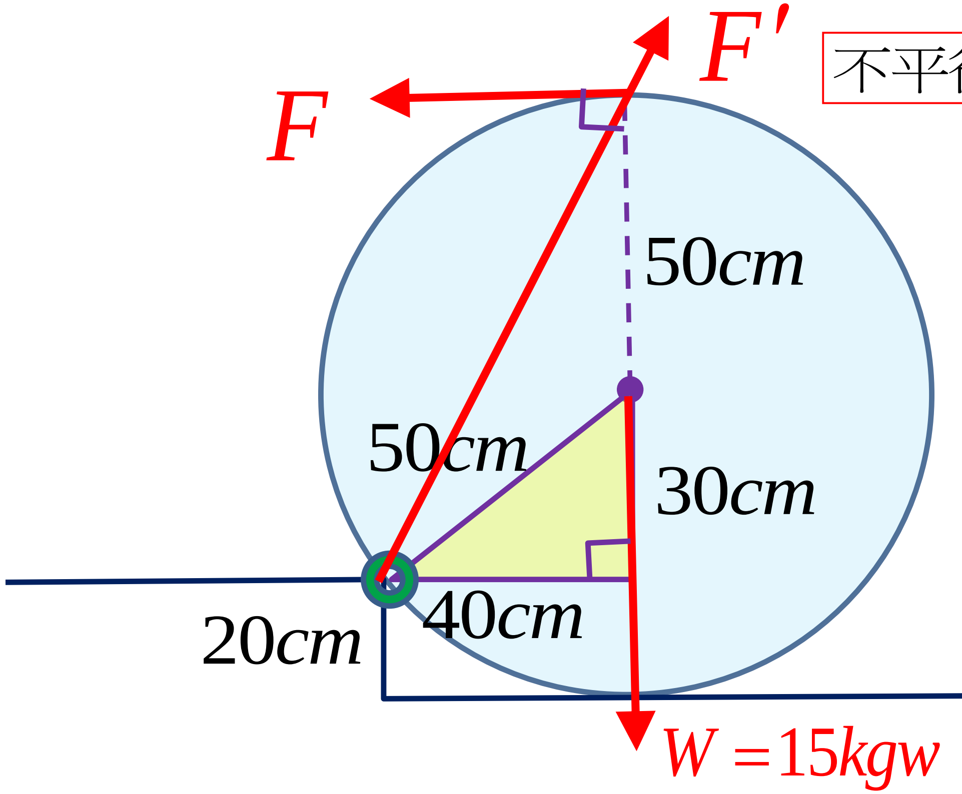
[結論]

不平行四力平衡　不一定會共點

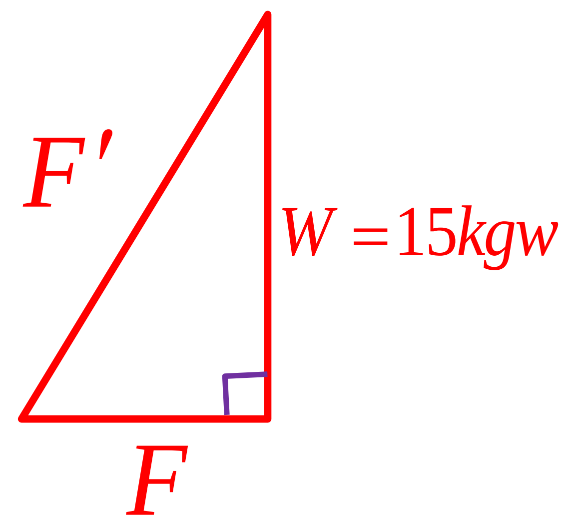
若四力中三力已共點　則四力必共點

[解析]

（3）



不平行三力平衡必交於一點



以階緣為轉軸　合力矩$=0$

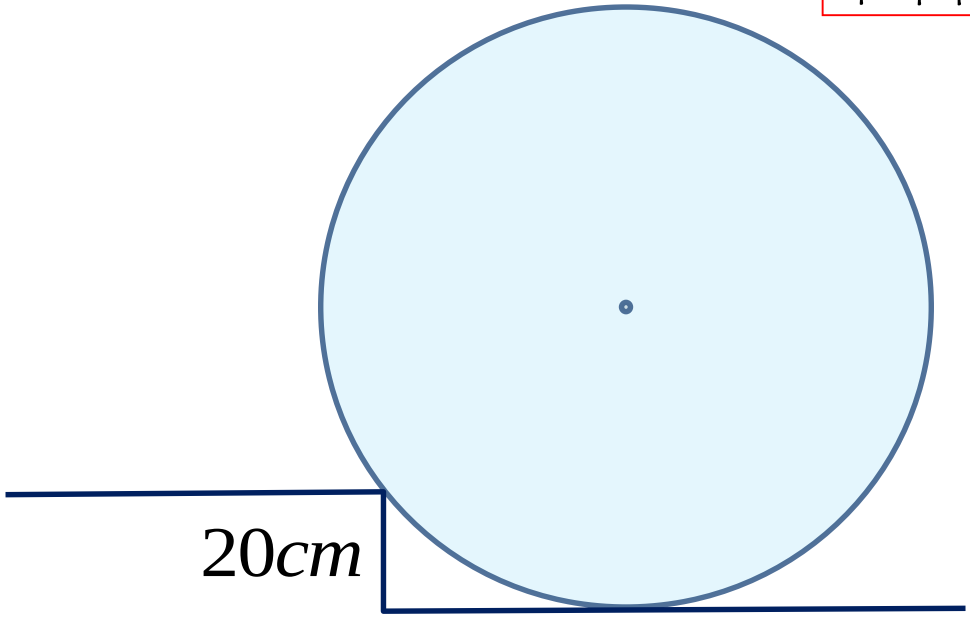
$15\times 40=F\times(30+50)\quad \therefore F=7.5$

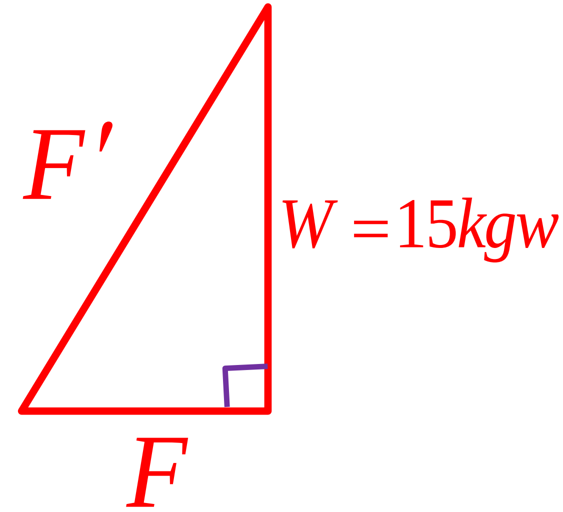
$\therefore$階緣作用力$F'=\sqrt{F^2+W^2}=\sqrt{7.5^2+15^2}=7.5\sqrt{5}$

[解析]

（3）

不平行三力平衡必交於一點





以階緣為轉軸　合力矩$=0$

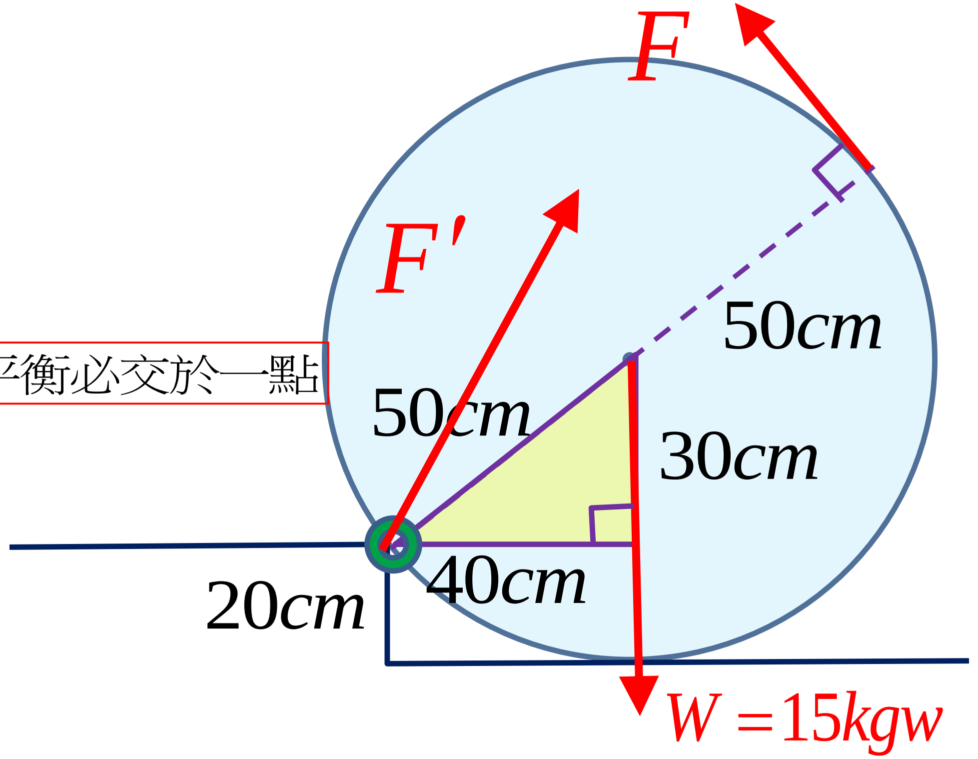
$15\times 40=F\times(30+50)$　$\therefore F=7.5$

$\therefore$階緣作用力$F'=\sqrt{F^2+W^2}=\sqrt{7.5^2+15^2}=7.5\sqrt{5}$

[解析]

（4）

不平行三力平衡必交於一點



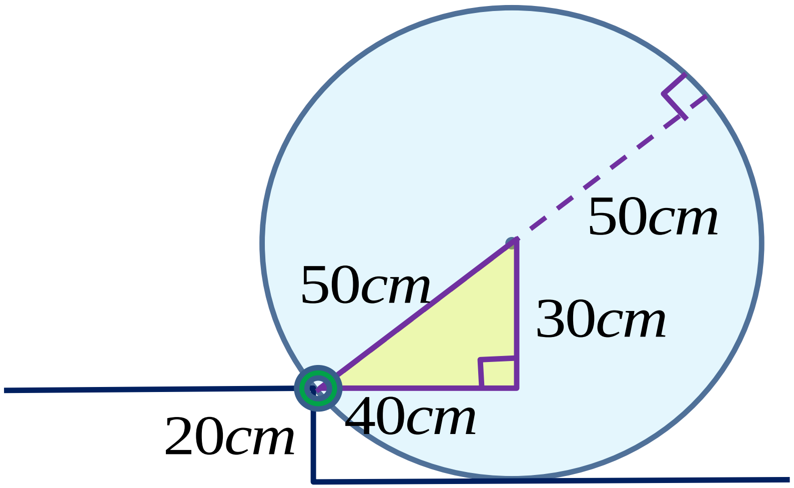
當力臂最長等於直徑時，力最小

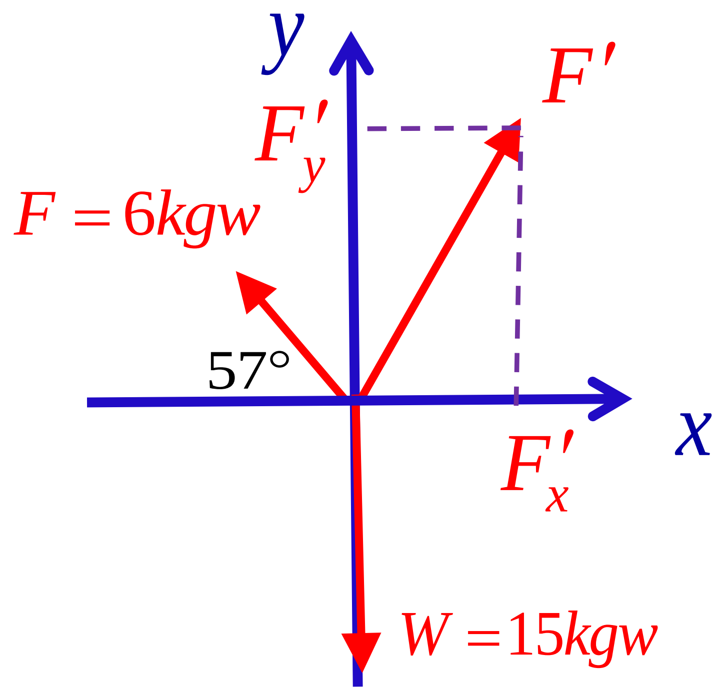
以階緣為轉軸　合力矩＝0

$$15\times 40=F\times(50+50)\quad \therefore F=6$$

[解析]

（4）

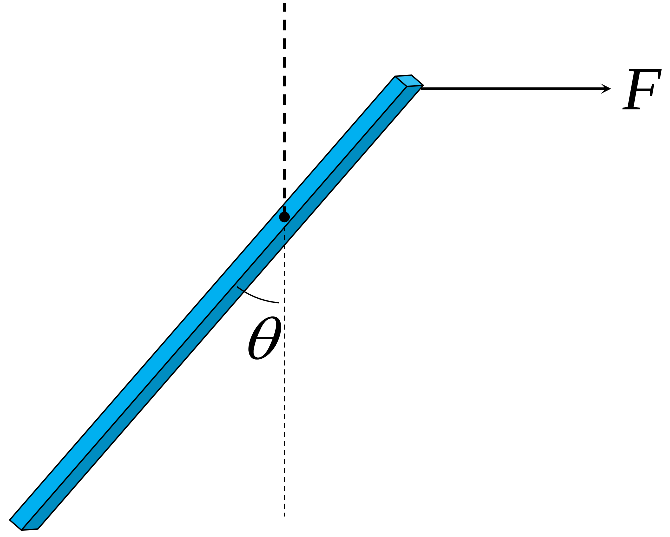




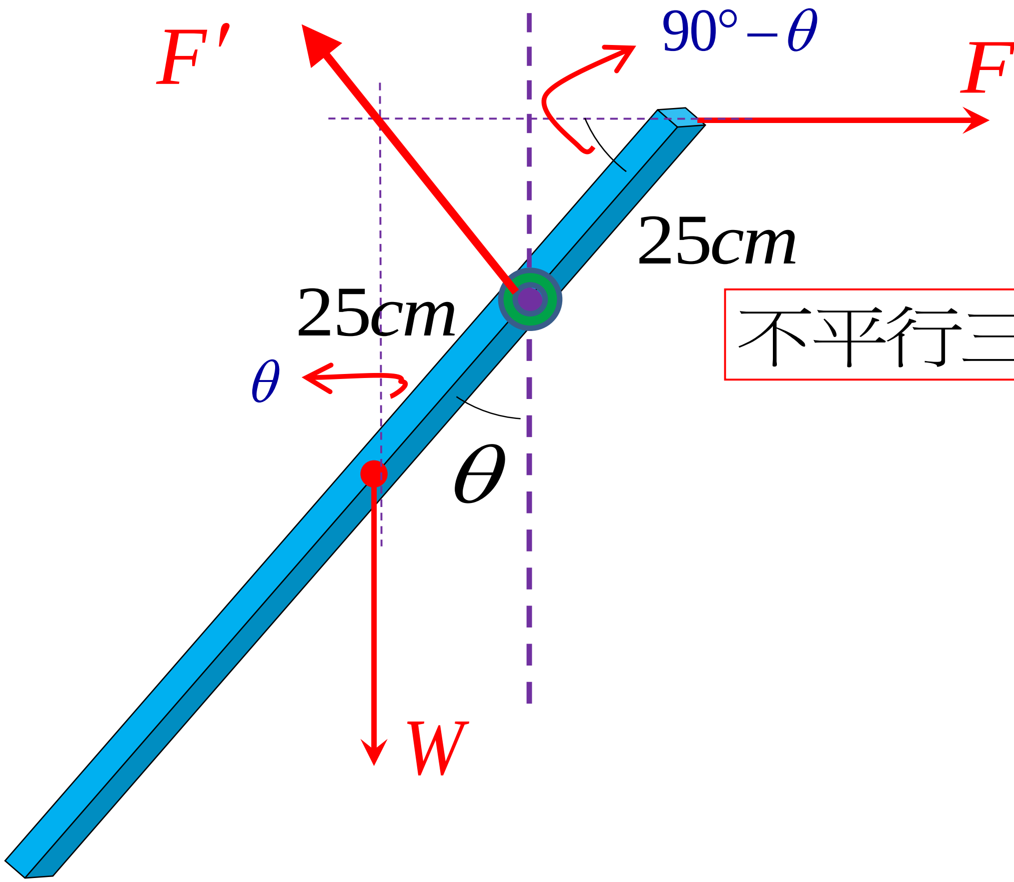
$$\text{合力}=0\ \begin{cases} x: F'_x = 6\times\dfrac{3}{5} = \dfrac{18}{5} \\ y: F'_y + 6\times\dfrac{4}{5} = 15 \rightarrow F'_y = \dfrac{51}{5} \end{cases}$$

$$\therefore F' = \sqrt{\left(F'_x\right)^2 + \left(F'_y\right)^2} = \sqrt{\left(\frac{18}{5}\right)^2 + \left(\frac{51}{5}\right)^2} = 3\sqrt{13}$$

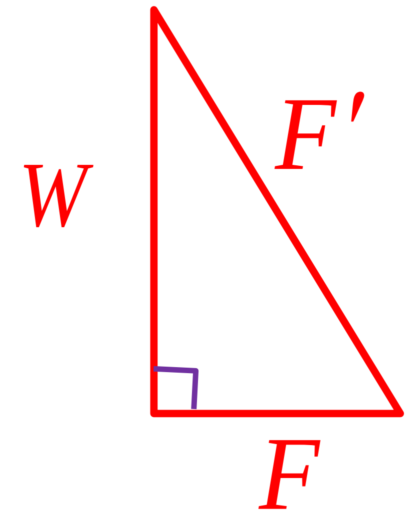
1.一均勻細桿，長*1*公尺，重量為*W*，在距離其上端*25*公分處以一釘子將此細桿釘在鉛直牆面上，使細桿可繞此釘子無摩擦地轉動。今施一水平力F於其上端，使細桿偏離鉛線θ角(θ< *90*°)，如圖所示，則在平衡時F量值為何？釘子作用在細桿上之力量值為？
(均以*W*、θ表示)



[解析]



不平行三力平衡必交於一點



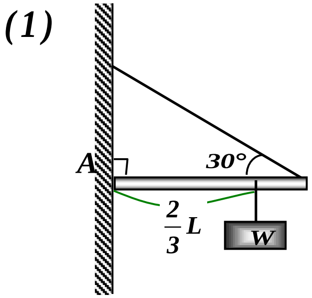
以階緣為轉軸　合力矩 $=0$

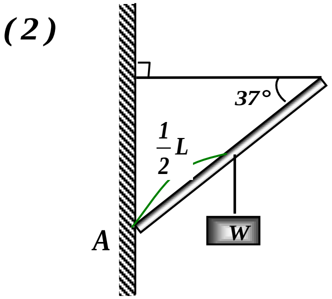
$25\times \mathrm{F}\times \sin(90°-\theta)=25\times \mathrm{W}\times \sin\theta \quad \therefore F=W\tan\theta$

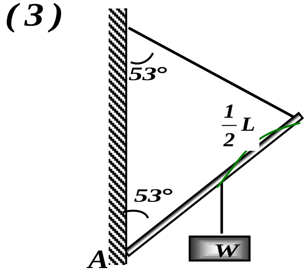
$\therefore$ 階緣作用力 $F'=\sqrt{F^2+W^2}=\sqrt{W\tan\theta^2+W^2}=W\sqrt{\tan\theta^2+1}$

$=W\sec\theta$

求下列三圖情形下，棒重不計長度*L*，棒上所掛物體重量*W*，木棒平衡時圖中繩的張力*T*及牆作用於棒的作用力F，分別為若干？(均以*W*表示)

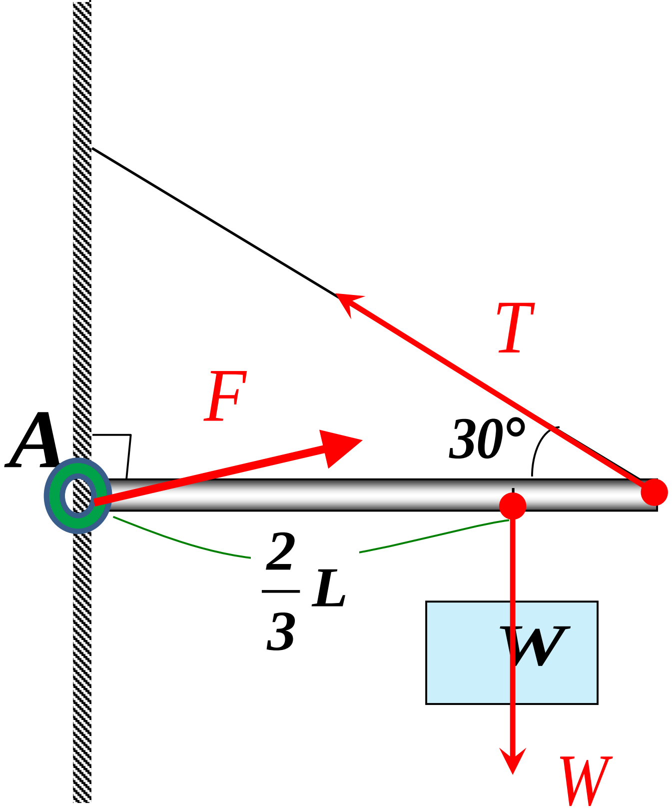






[解析]

(1)

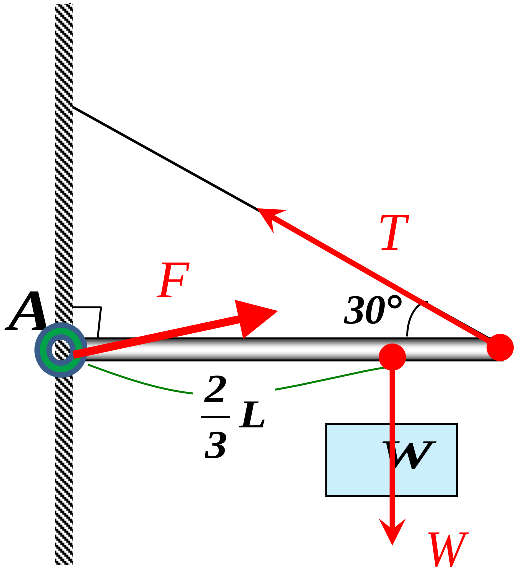


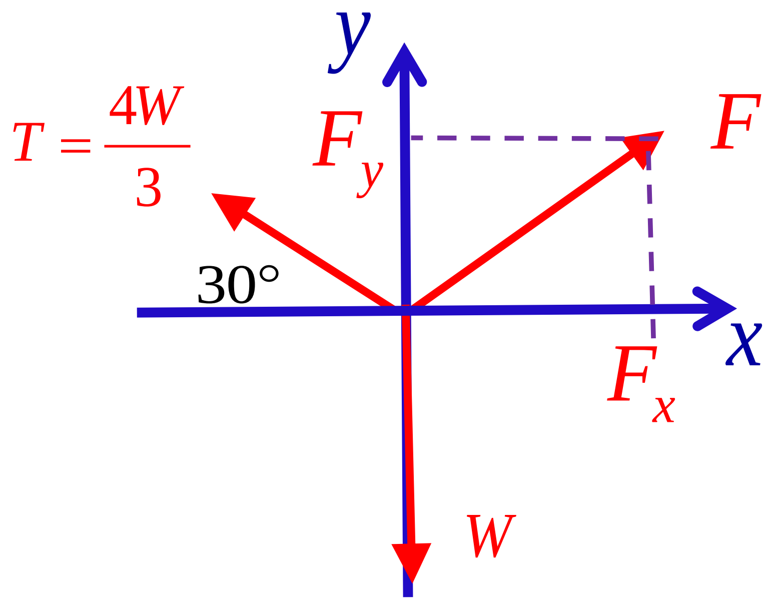
以A為轉軸 合力矩＝0

$$L \times T \times \sin 30^\circ = \frac{2L}{3} \times W \qquad \therefore T = \frac{4W}{3}$$

[解析]

(1)



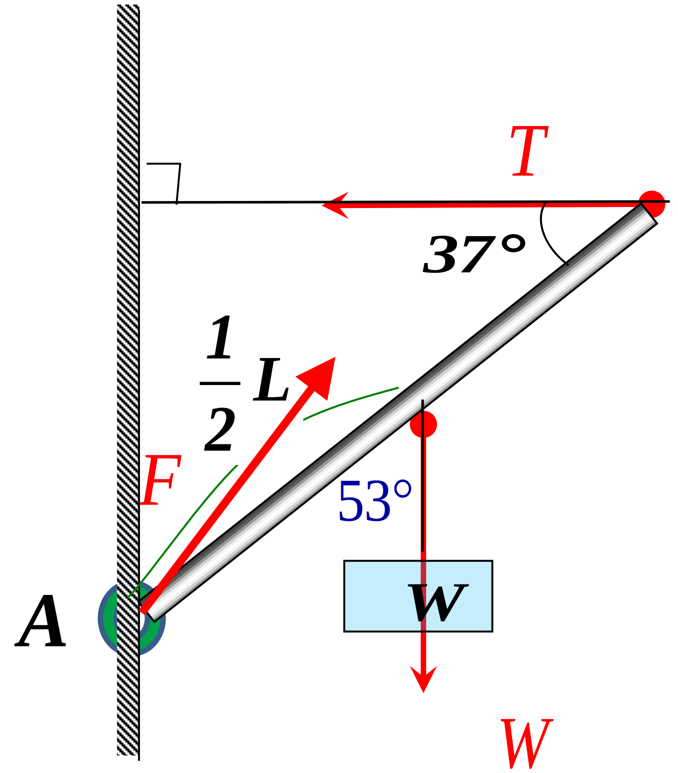


$$\text{合力}=0\begin{cases} x: F_x=\dfrac{4W}{3}\times\dfrac{\sqrt{3}}{2}=\dfrac{2\sqrt{3}}{3}W \\ y: F_y+\dfrac{4W}{3}\times\dfrac{1}{2}=W\rightarrow F_y=\dfrac{1}{3}W \end{cases}$$

$$\therefore F=\sqrt{(F_x)^2+(F_y)^2}=\sqrt{\left(\frac{2\sqrt{3}}{3}W\right)^2+\left(\frac{1}{3}W\right)^2}=\frac{\sqrt{13}}{3}W$$

[解析]

(2)

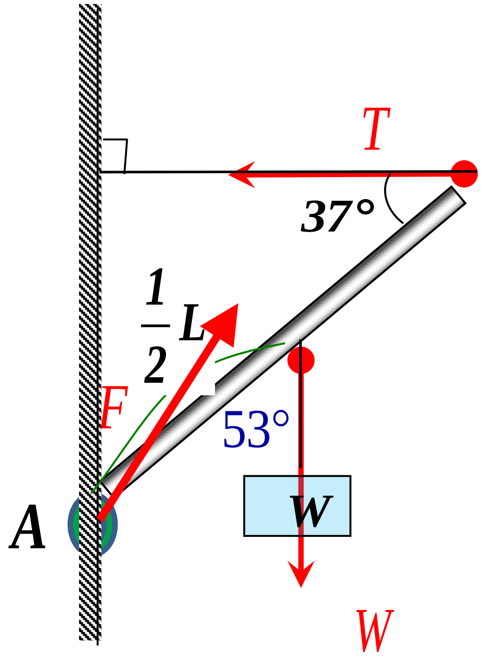


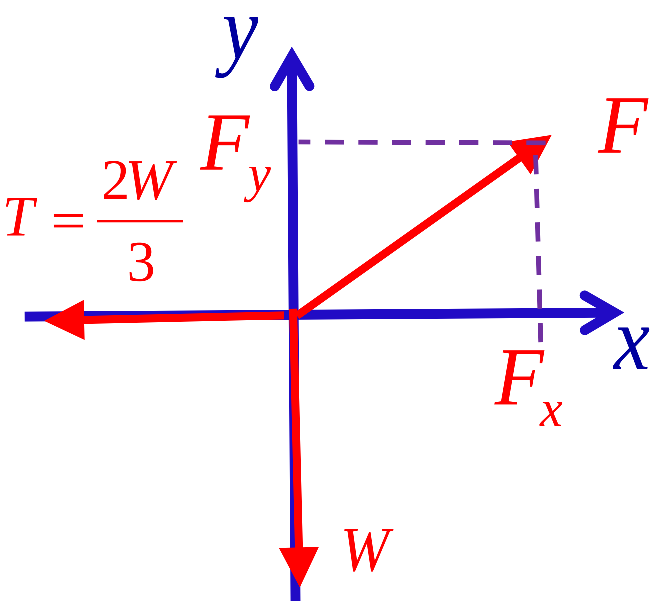
以A為轉軸　合力矩＝0

$$L \times T \times \sin 37° = \frac{L}{2} \times W \times \sin 53° \quad \therefore T = \frac{2W}{3}$$

[解析]

(2)



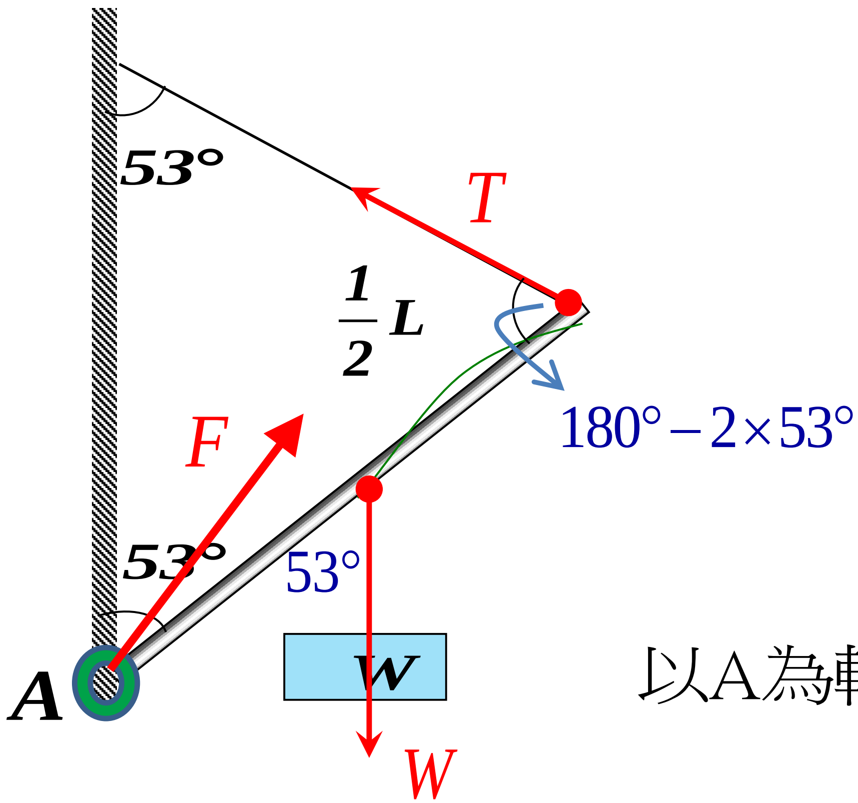


$$合力=0\begin{cases} x: F_x=\dfrac{2W}{3} \\ y: F_y=W \end{cases}$$

$$\therefore F=\sqrt{(F_x)^2+(F_y)^2}=\sqrt{\left(\frac{2}{3}W\right)^2+(W)^2}=\frac{\sqrt{7}}{3}W$$

[解析]

(3)



以A為轉軸　合力矩 = 0

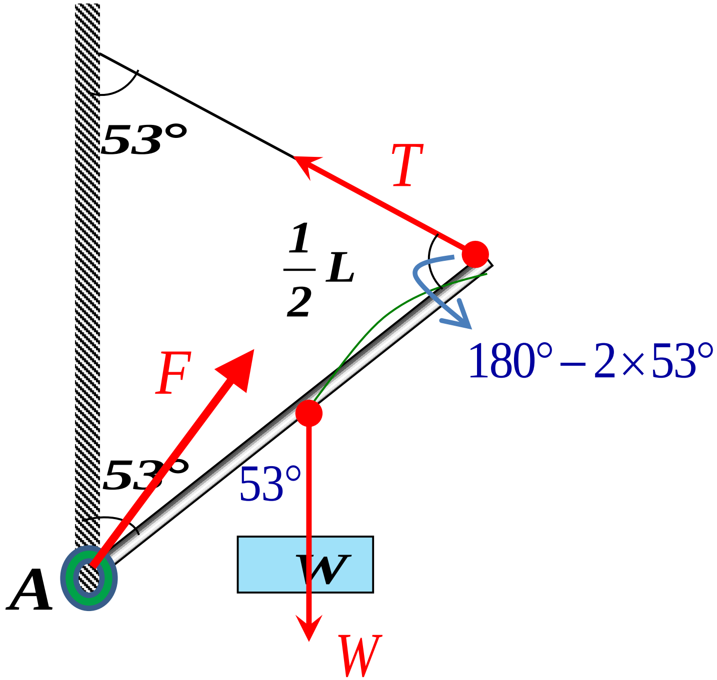
$$L \times T \times \sin\left(180^\circ - 2 \times 53^\circ\right) = \frac{L}{2} \times W \times \sin 53^\circ$$

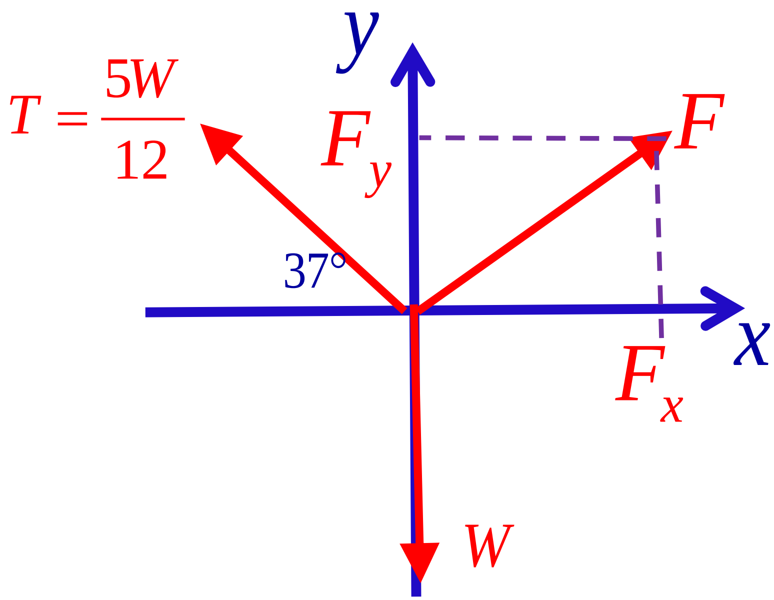
$$\because \sin\left(180^\circ - 2 \times 53^\circ\right) = 2\sin 53^\circ \cos 53^\circ$$

$$\therefore T = \frac{5W}{12}$$

[解析]

(3)



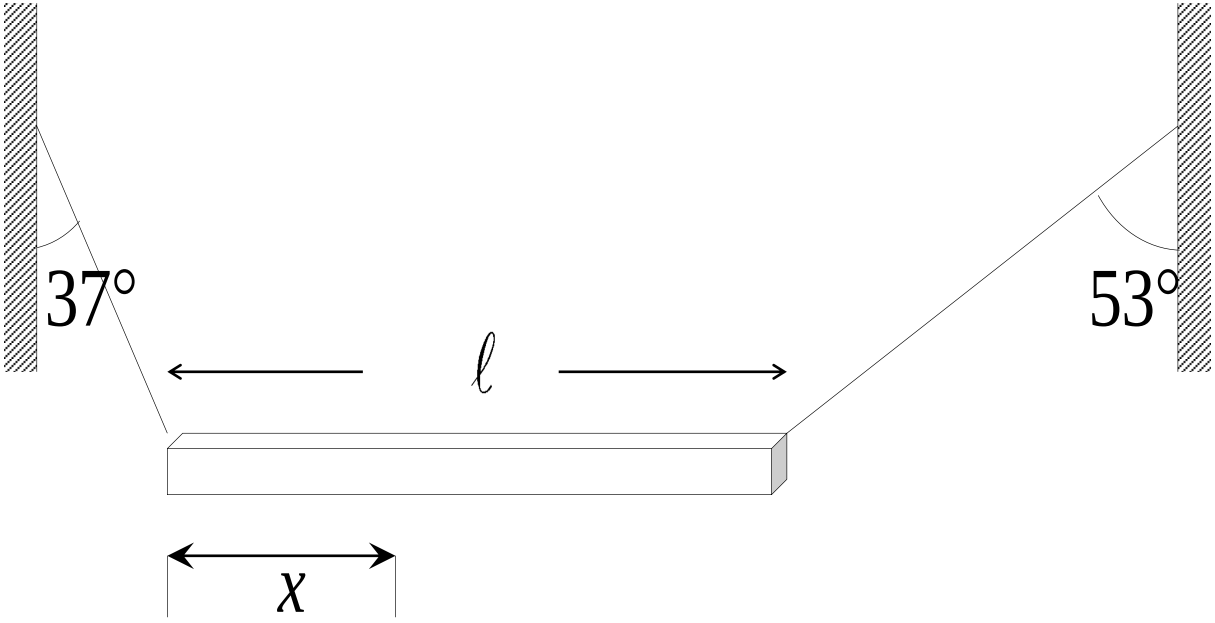


$$\text{合力}=0\ \begin{cases} x: F_x = \dfrac{5W}{12}\times\dfrac{4}{5} = \dfrac{W}{3} \\ y: F_y + \dfrac{5W}{12}\times\dfrac{3}{5} = W \rightarrow F_y = \dfrac{3W}{4} \end{cases}$$

$$\therefore F = \sqrt{\left(F_x\right)^2 + \left(F_y\right)^2} = \sqrt{\left(\frac{W}{3}\right)^2 + \left(\frac{3W}{4}\right)^2} = \frac{\sqrt{97}}{12}W$$

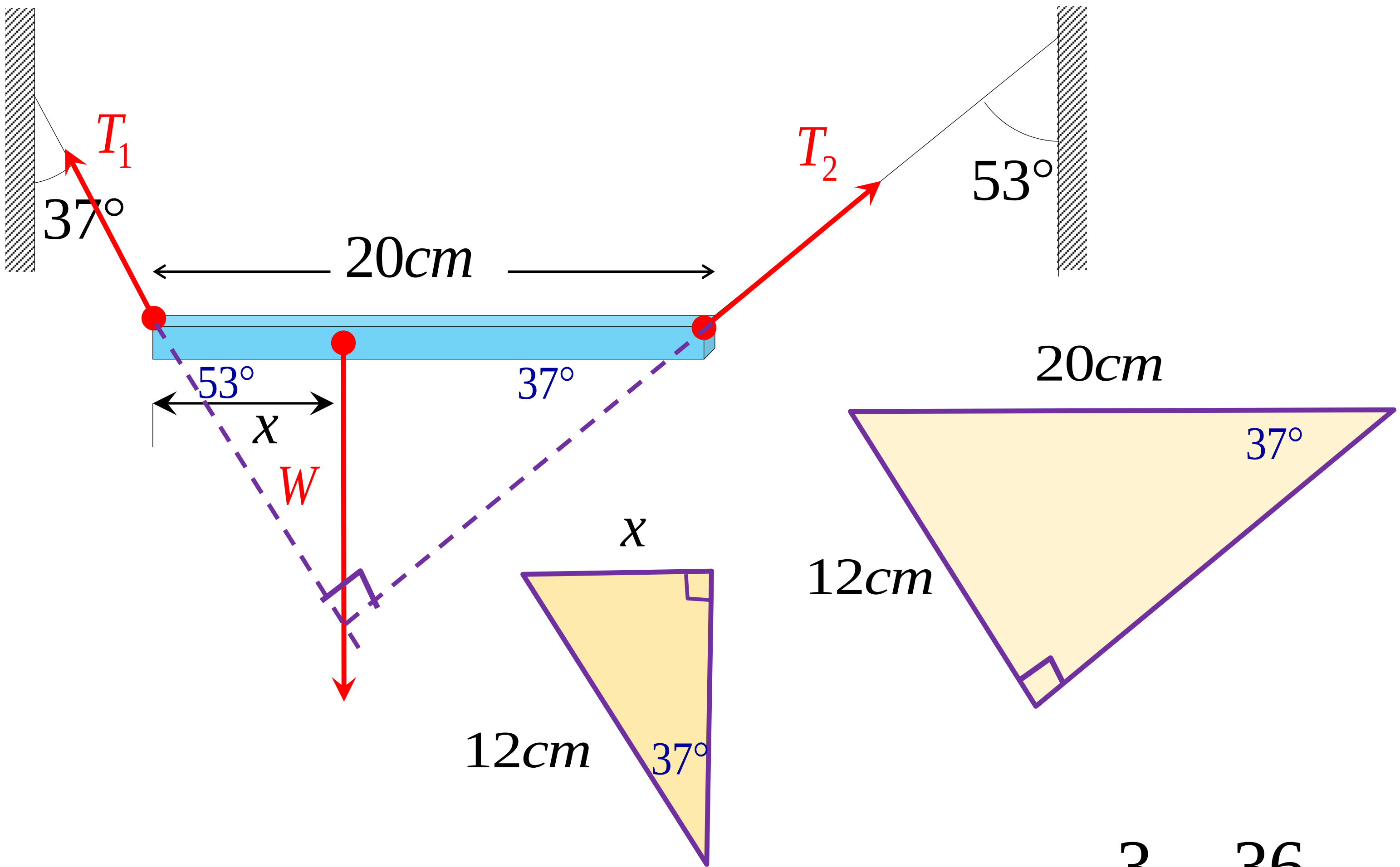
第109頁

1.一不均勻之桿重*W*，由二輕繩懸掛於水平位置，如圖，一繩與牆之夾角*37*°，另一角度為 *53*°，若桿長 l= *20*公分，則重心到左端之距離*x*為多少？



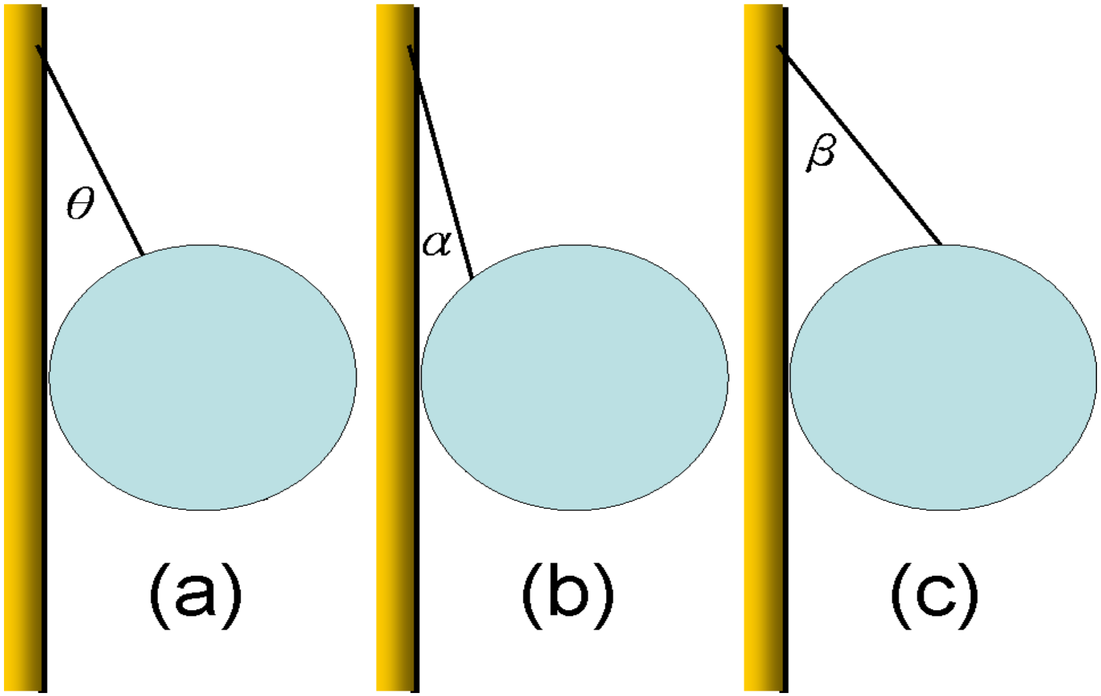
[解析]

$\boxed{\text{不平行三力平衡必交於一點}}$

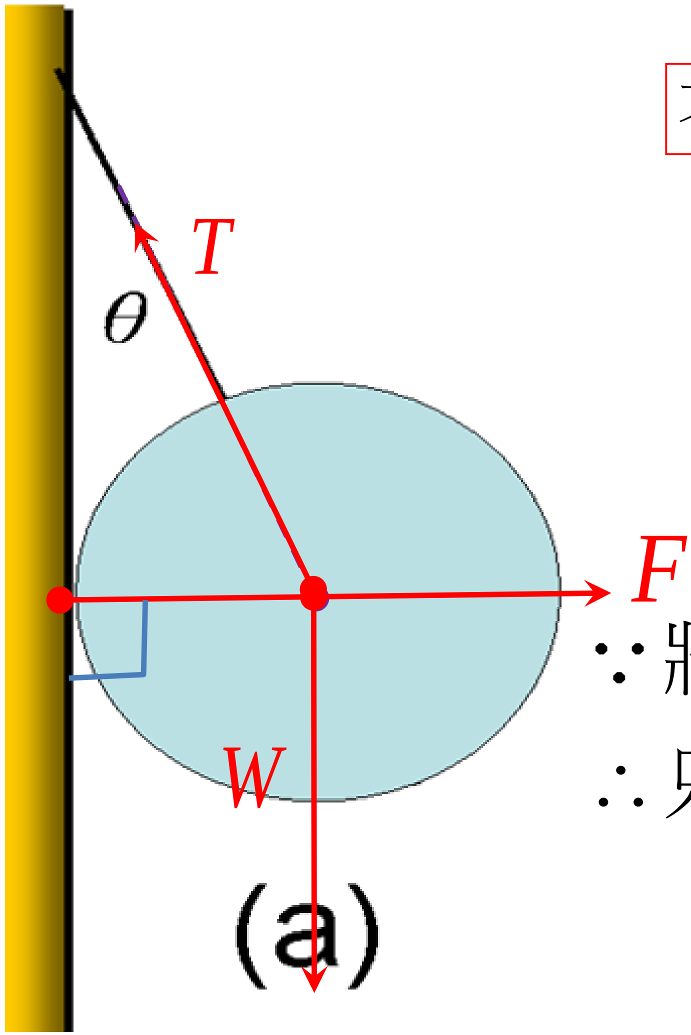


$$\therefore x = 12 \times \frac{3}{5} = \frac{36}{5}$$

2.圖中，三均勻的球重W，各以細線懸吊於牆壁上，線與牆壁的夾角各為θ、α和β，(a)圖中的細線方向通過球心，(b)和(c)圖則否，問：三圖中，牆壁與球之間有摩擦力嗎？方向向哪裡？



[解析]

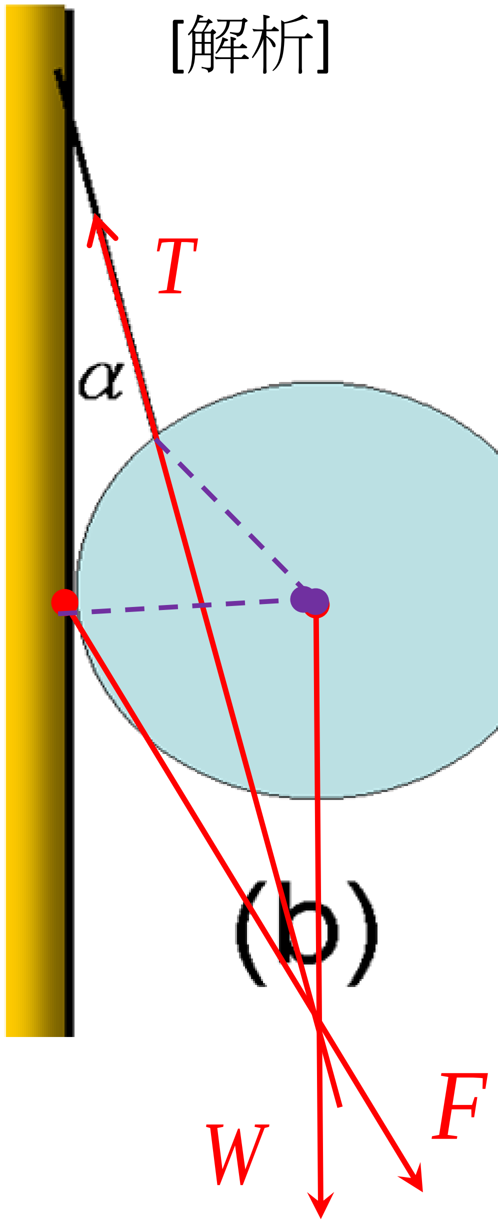


不平行三力平衡必交於一點

∵牆壁與球間作用力垂直牆壁

∴只有正向力，沒有摩擦力

[解析]



不平行三力平衡必交於一點

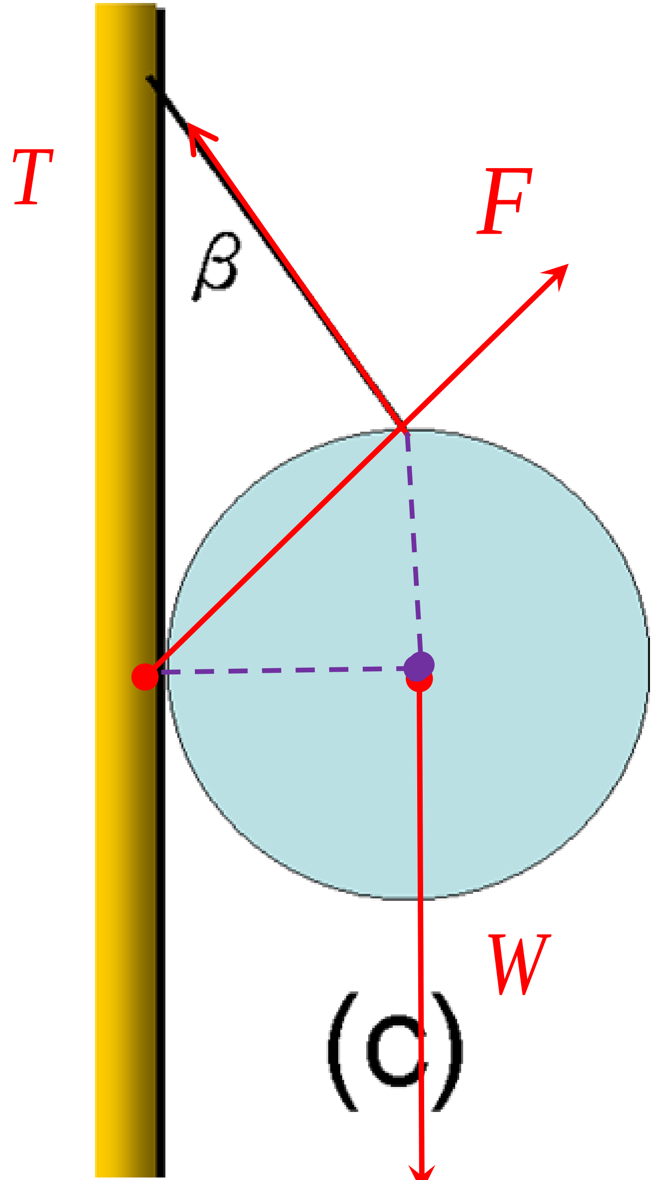
∵牆壁與球間作用力不垂直牆壁

∴垂直牆壁方向分量為有正向力

平行牆壁方向分量有摩擦力

沿牆壁向下

[解析]



不平行三力平衡必交於一點

∵牆壁與球間作用力不垂直牆壁

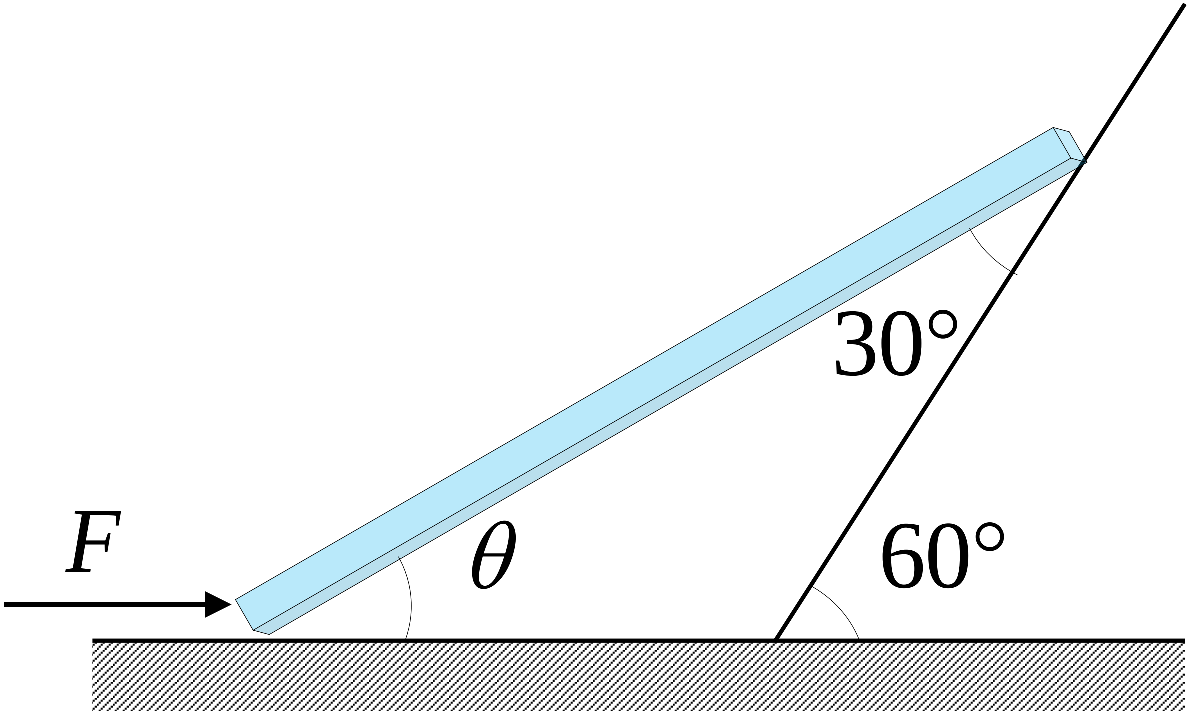
∴垂直牆壁方向分量為有正向力

平行牆壁方向分量有摩擦力

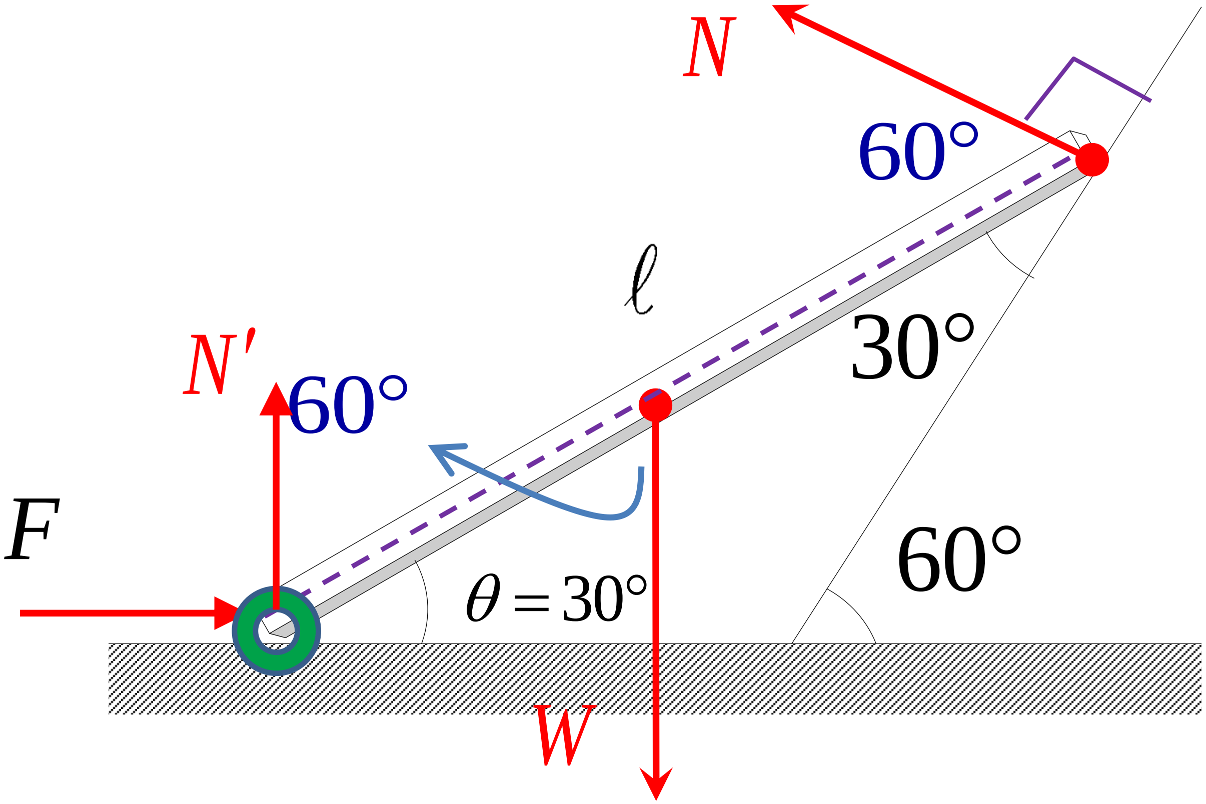
沿牆壁向上

第110頁

1.重W的均勻木棒斜靠在光滑的牆及地面上，欲保持靜止不動，則水平施力F（作用於木棒最下端）的大小為多少？
地面與木棒作用力？



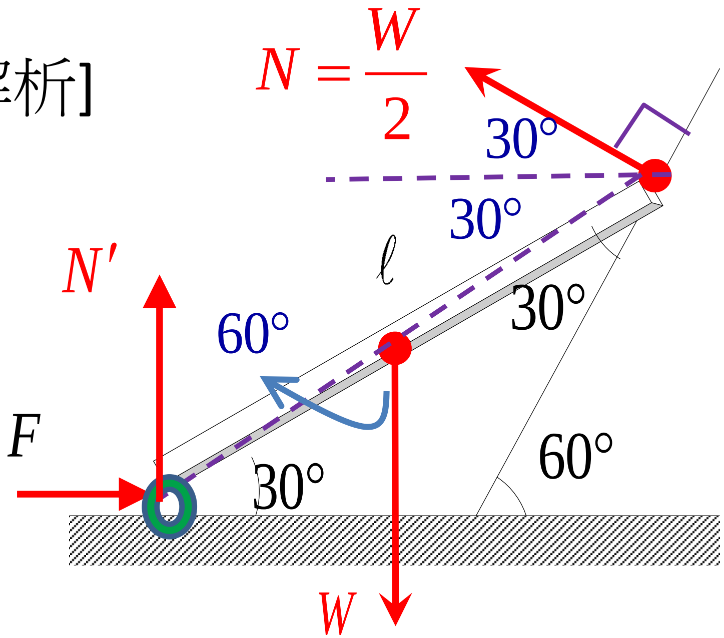
[解析]

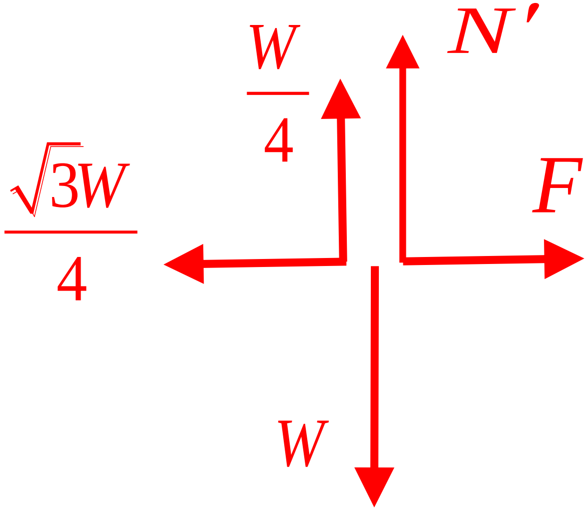


以觸地端為轉軸　合力矩 = 0

$$\ell \times N \times \sin 60° = \frac{\ell}{2} \times W \times \sin 60° \quad \therefore N = \frac{W}{2}$$

[解析]

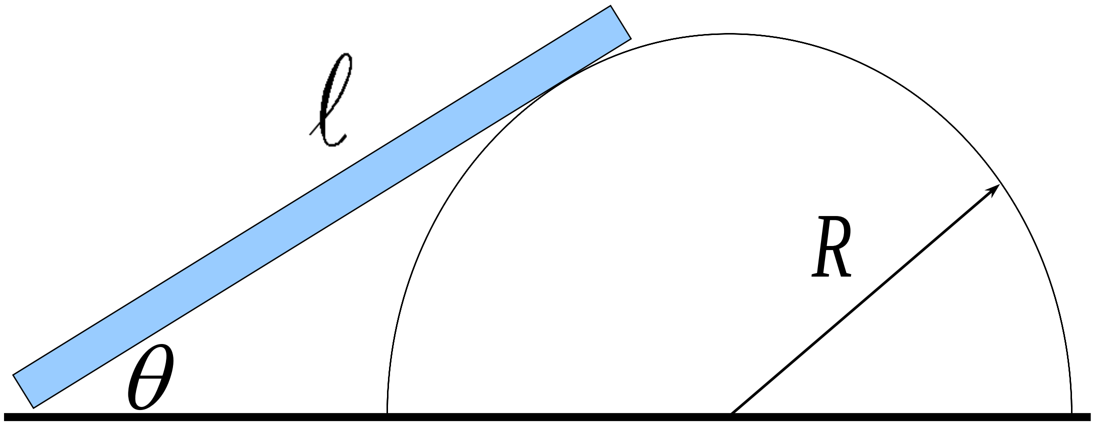




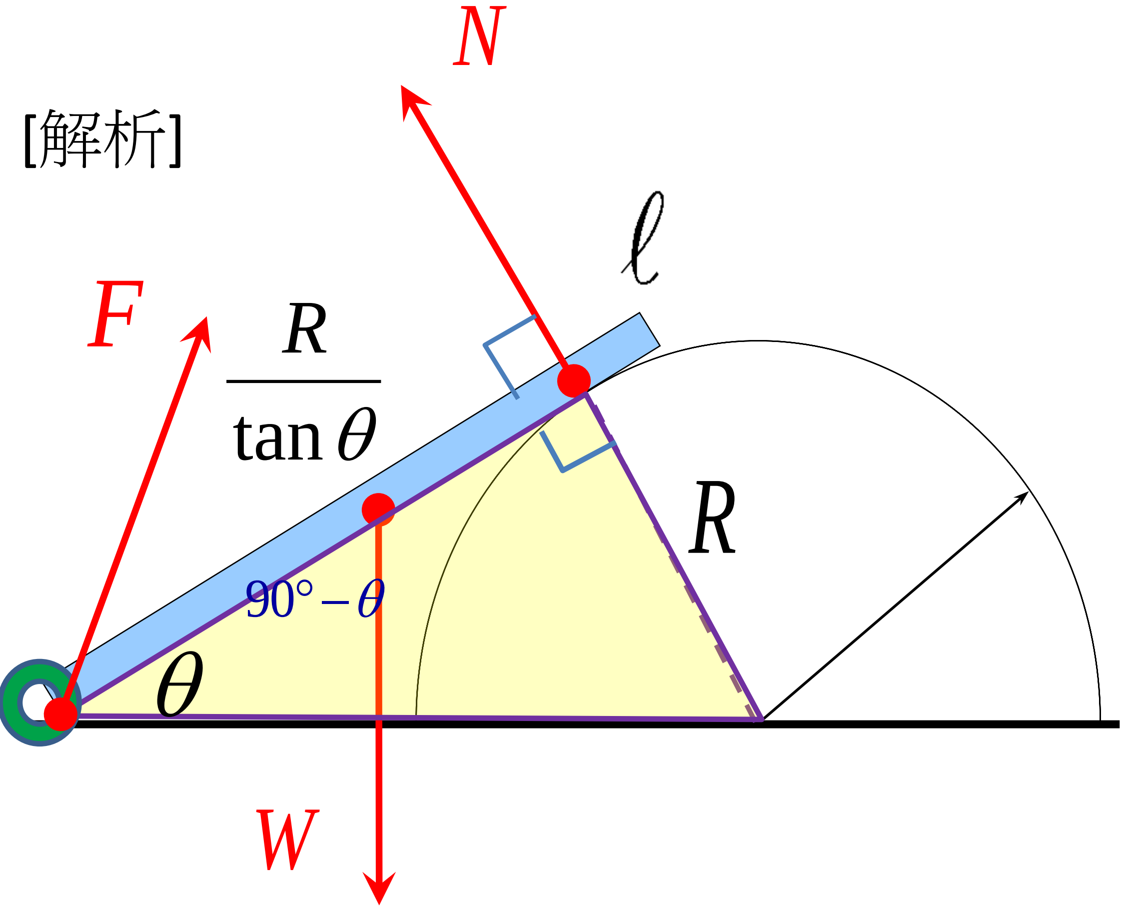
$$合力=0\quad\begin{cases} x: F=\dfrac{\sqrt{3}W}{4} \\ y: N'+\dfrac{W}{4}=W \rightarrow N'=\dfrac{3W}{4} \end{cases}$$

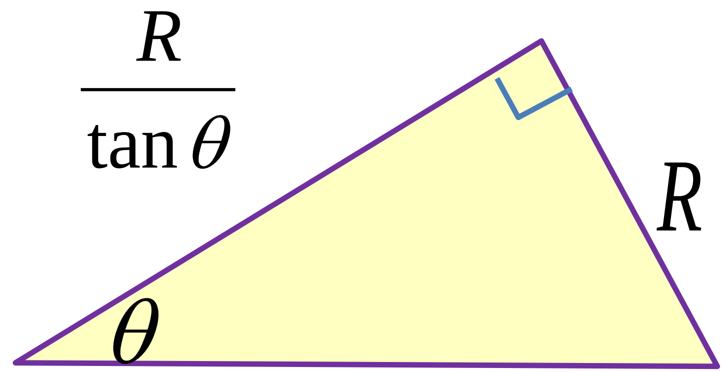
2. 圖中，一均勻木棒重*W*，長度為l，斜靠在半徑為*R*的光滑固定半球面上，棍與粗糙地面的夾角ϑ為30°，若l=2*R*則，

(*a*)球面對木棍的作用力？(*b*)地面對木棍的正向力？



[解析]

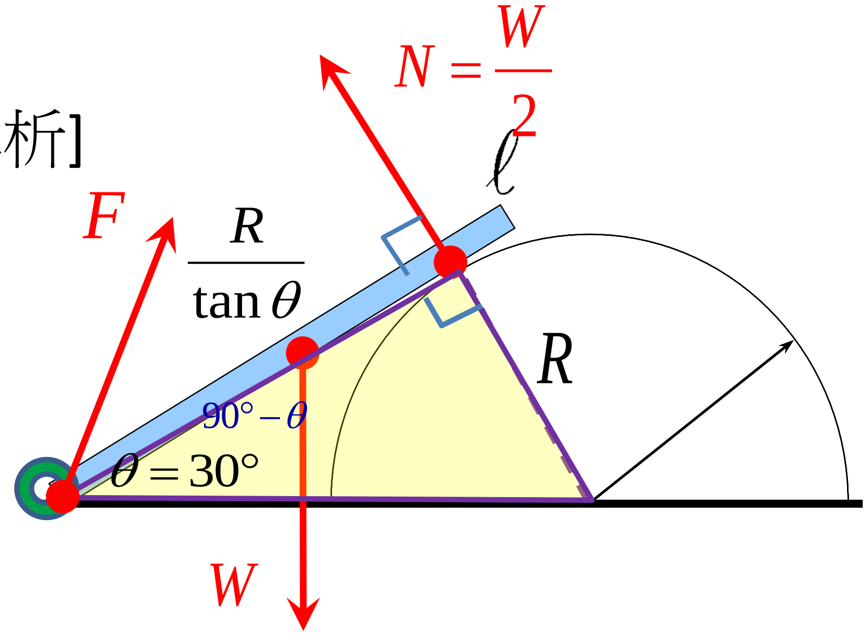


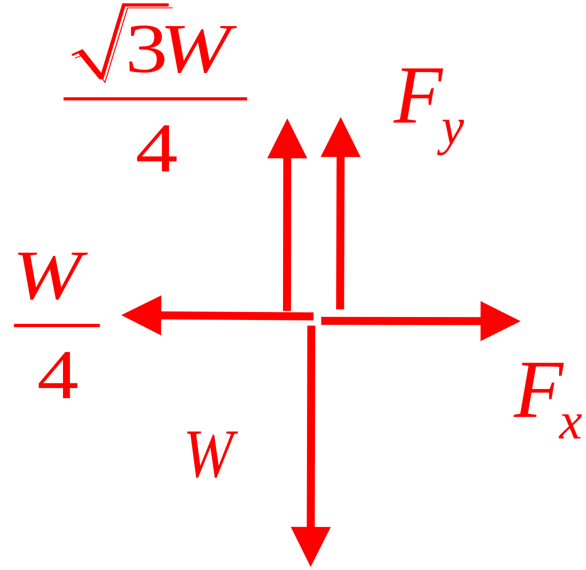


以觸地端為轉軸　合力矩 $=0$

$$N\times\frac{R}{\tan\theta}=\frac{\ell}{2}\times W\times\sin\left(90^\circ-\theta\right)\qquad\therefore N=\frac{\ell W}{2R}\sin\theta=\frac{W}{2}$$

[解析]

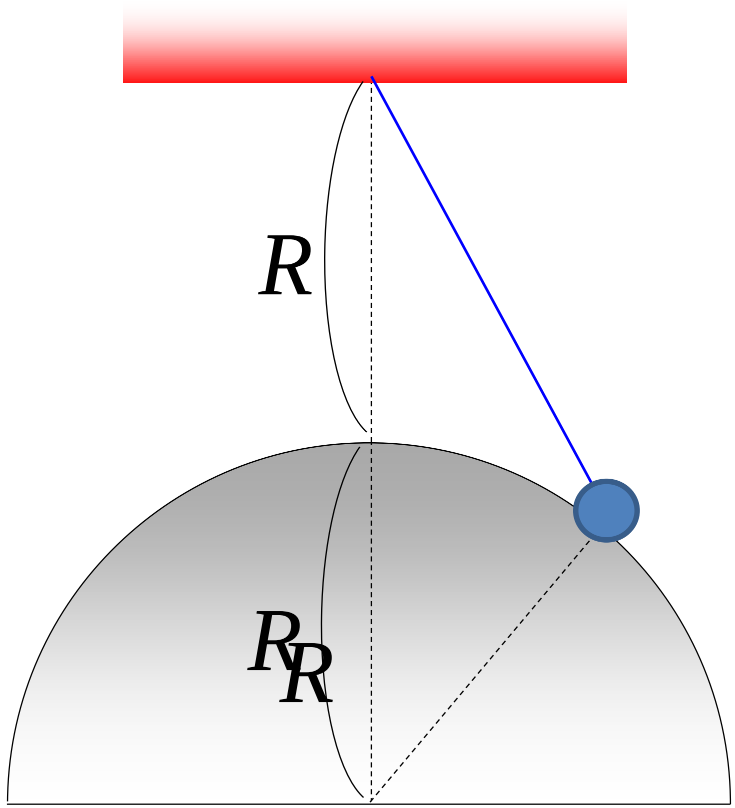


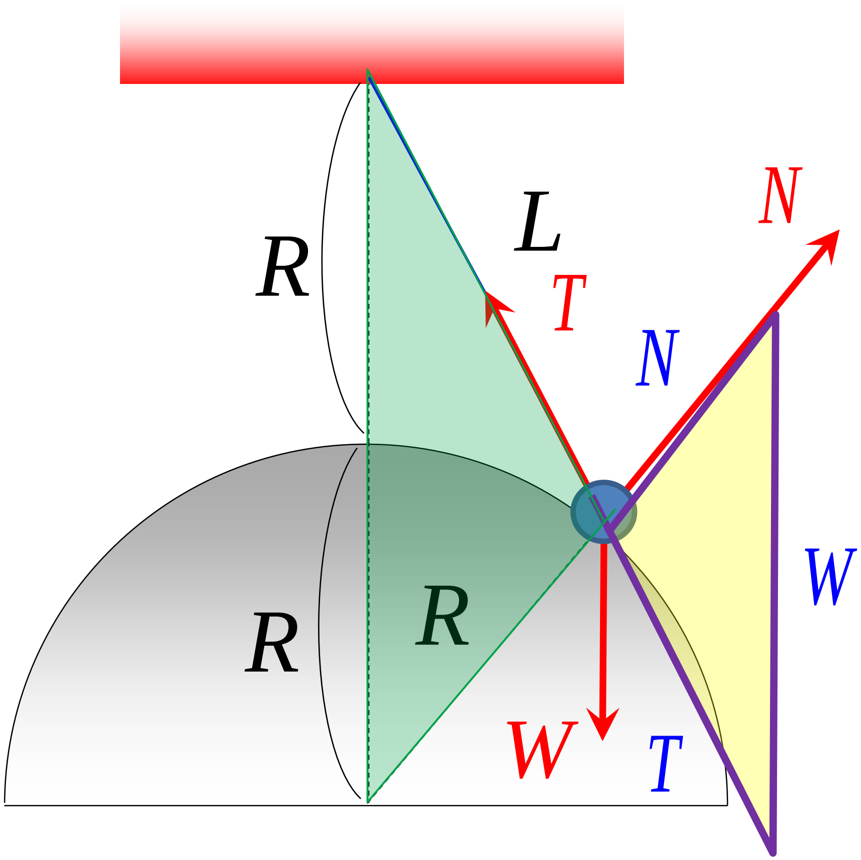


$$
合力=0\begin{cases} x: F_x=\frac{W}{4} \\ y: F_y+\frac{\sqrt{3}W}{4}=W \rightarrow F_y=\frac{4-\sqrt{3}W}{4} \end{cases}
$$

第111頁

1. 圖示，用長為 *L* 的細繩吊一重量為 *W* 的小球(可視為質點)，放在表面光滑半徑為 R 的半圓柱上；細繩的懸點在柱面中心的正上方，距柱面頂點為 R，則平衡時柱面對球的正向力量值？

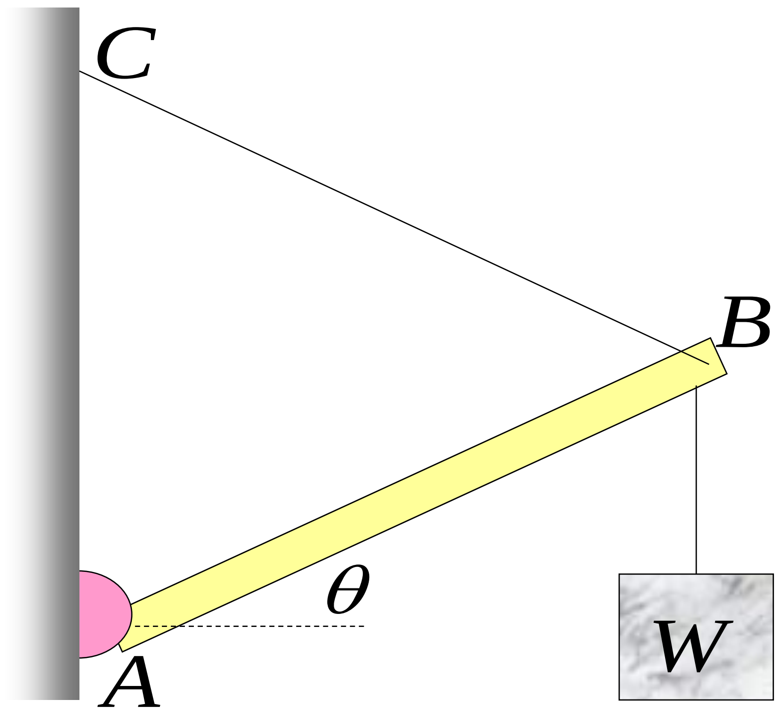




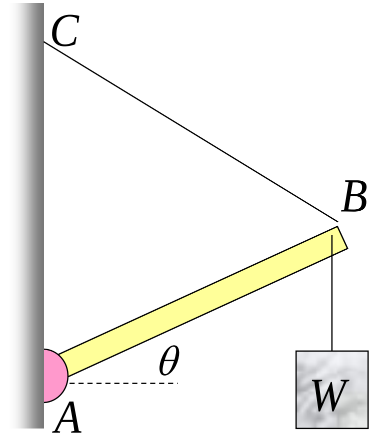
相似形

$$\frac{W}{2R}=\frac{N}{R}=\frac{T}{L}\therefore N=\frac{W}{2}$$

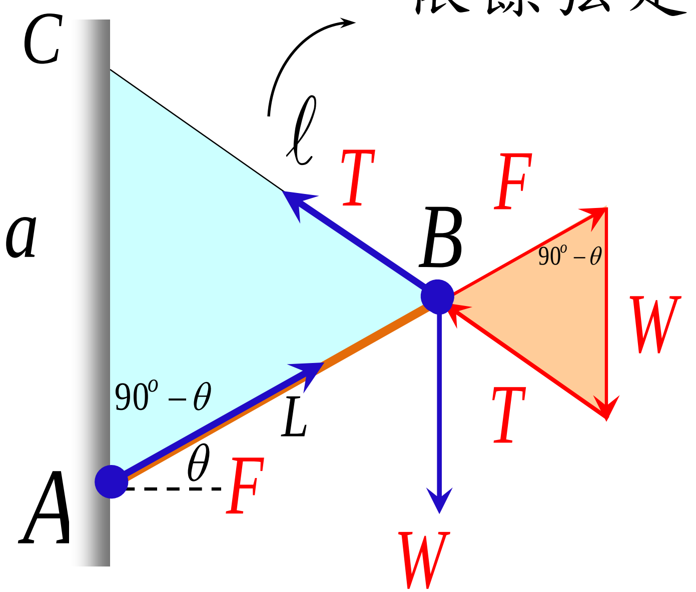
2.圖中，一質量可忽略不計的木棒以*A*點作為樞紐可繞鉛直面轉動，另一端頂住懸掛重物*W*的細繩。系統平衡時，木棒與水平方向之夾角為θ ，若木棒長度為*L*，圖中樞紐*A*到*C*點的長度為*a*，則細繩對木棒的作用力為何？



[解析]



[提示]　可使用相似形比較三力大小關係。



依餘弦定理 $\ell = \sqrt{a^2 + L^2 - 2aL\cos(90^o - \theta)}$

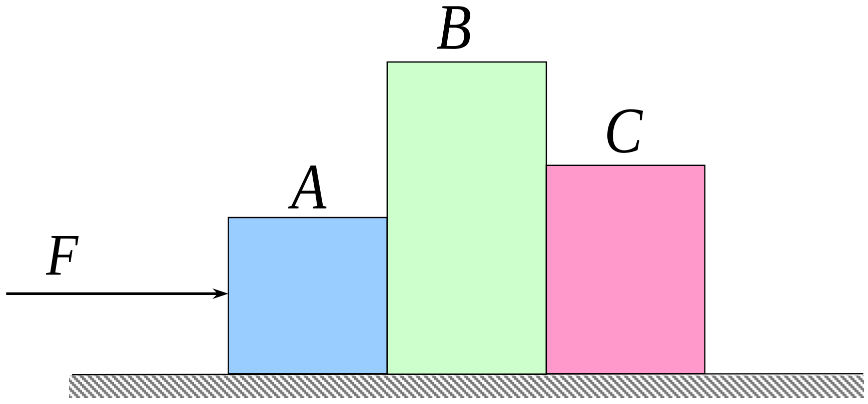
$= \sqrt{a^2 + L^2 - 2aL\sin\theta}$

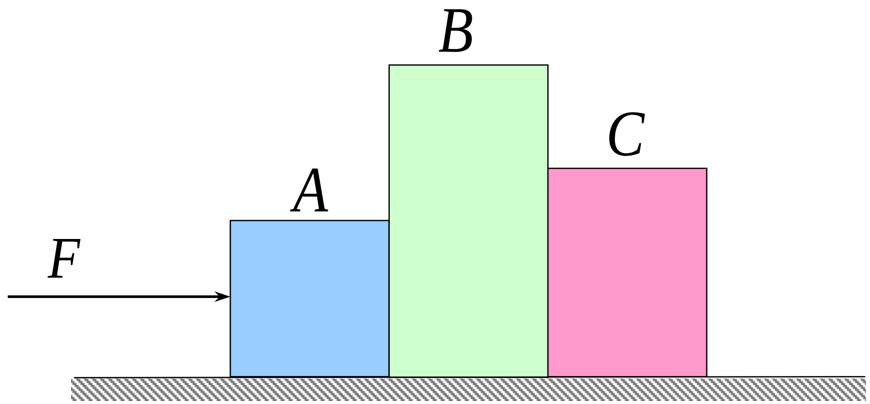
由相似三角形 $\dfrac{T}{W} = \dfrac{\sqrt{a^2 + L^2 - 2aL\sin\theta}}{a}$

$\therefore T = \dfrac{\sqrt{a^2 + L^2 - 2aL\sin\theta}}{a} W$

1.水平桌面上有*A*、*B*、*C*三木塊，其重量分別為2 kgw、6 kgw、4 kgw。已知木塊與水平面間的靜摩擦係數皆為0.5，今施一水平推力5 kgw於木塊*A*上，則

(*a*)地面與各個木塊間的摩擦力為何？(*b*)木塊兩兩間的作用力為何？



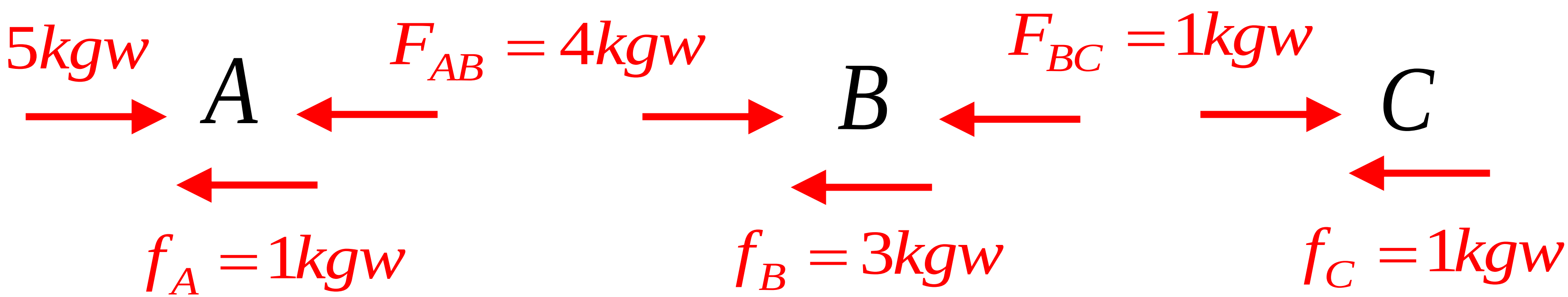


$$f_{AS}(\max) == \mu_S m_A g = 0.2 \times 2 = 1[kgw]$$

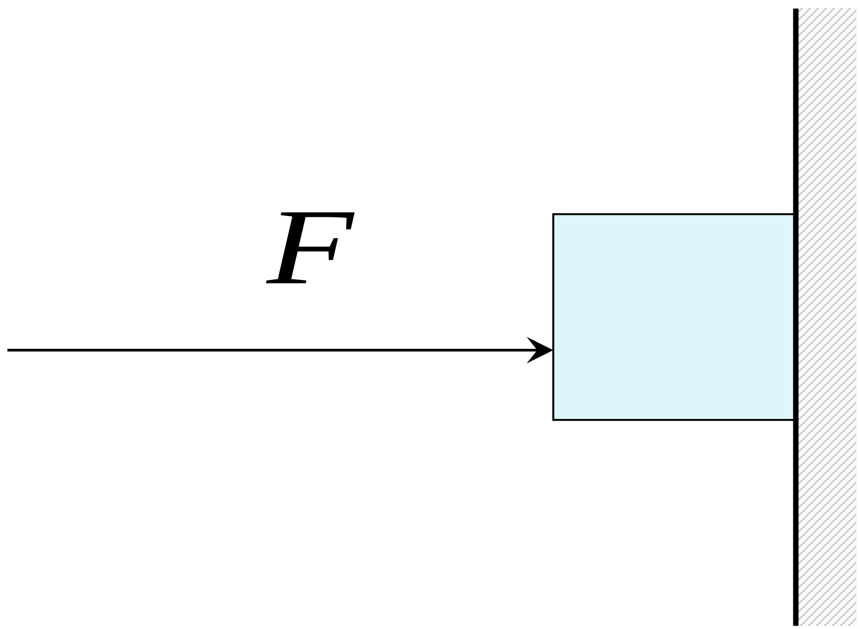
$$f_{BS}(\max) == \mu_S m_B g = 0.2 \times 6 = 3[kgw]$$

$$f_{CS}(\max) == \mu_S m_C g = 0.2 \times 4 = 2[kgw]$$

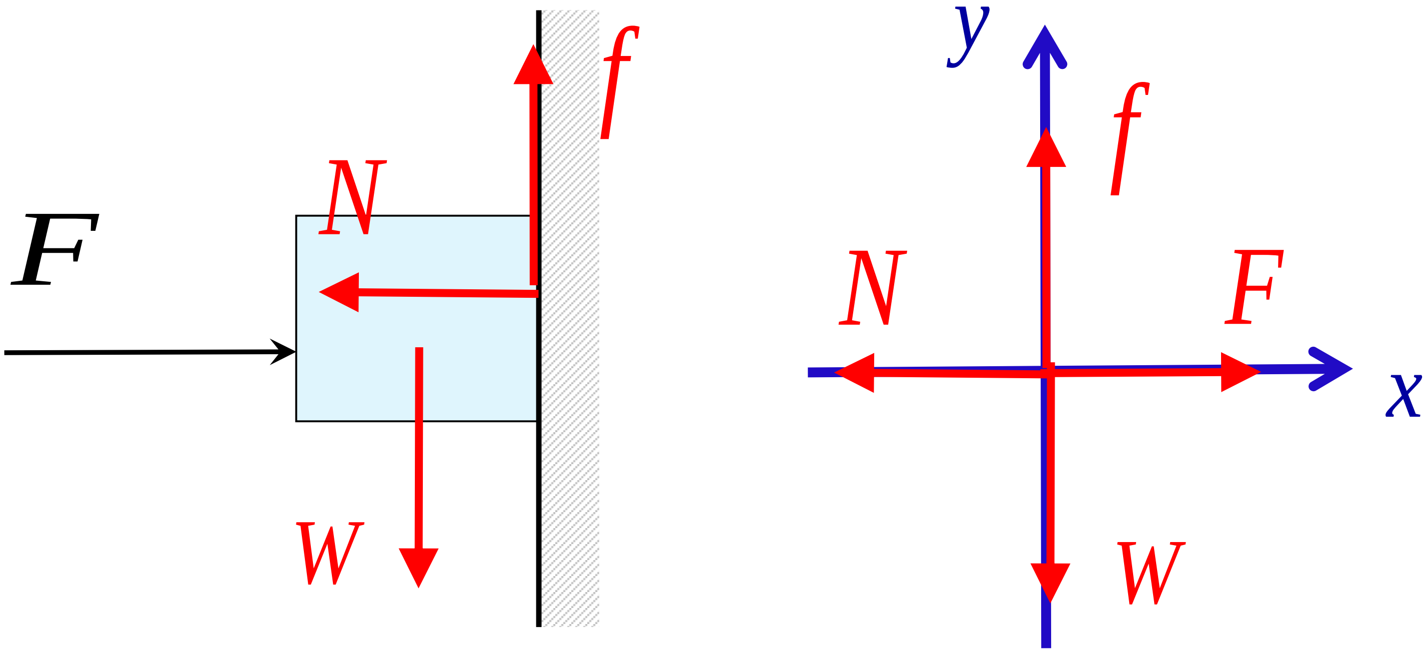
$\because 1+3+2=6>5$ $\therefore$ 靜止 推不動



2.如右圖，重量W的物體，靠於鉛直牆上與牆間之靜摩擦係數為 μ_S，今施一水平力F於物體，使物體不滑下，則（A）$F = W$（B）$W=\mu_S F$（C）牆與物體間之摩擦力為W（D）牆施於物體之淨力為 $F\sqrt{1+\mu_S{}^2}$（E）若物體與牆無摩擦，則F無論多大均不能平衡。



[解析]



$(A)(B)(C)$合力=0 $\begin{cases} x: N = F \\ y: f = W \end{cases}$

$\therefore f \leq f_{S(\max)} = \mu_S F$

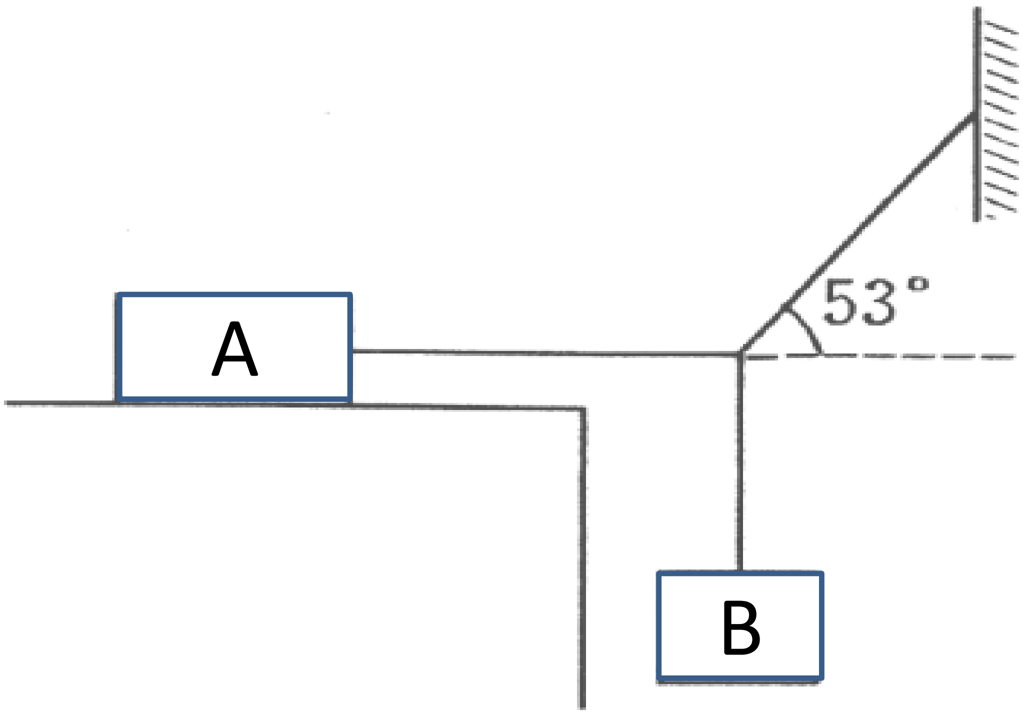
$(D)\sqrt{N^2 + f^2} = \sqrt{F^2 + W^2} \leq F\sqrt{1 + {\mu_S}^2}$

[練習題]

圖中，物體A重*10kgw*與平面間之靜摩擦係數為0.6 ， 此系統成平衡狀態，試問：

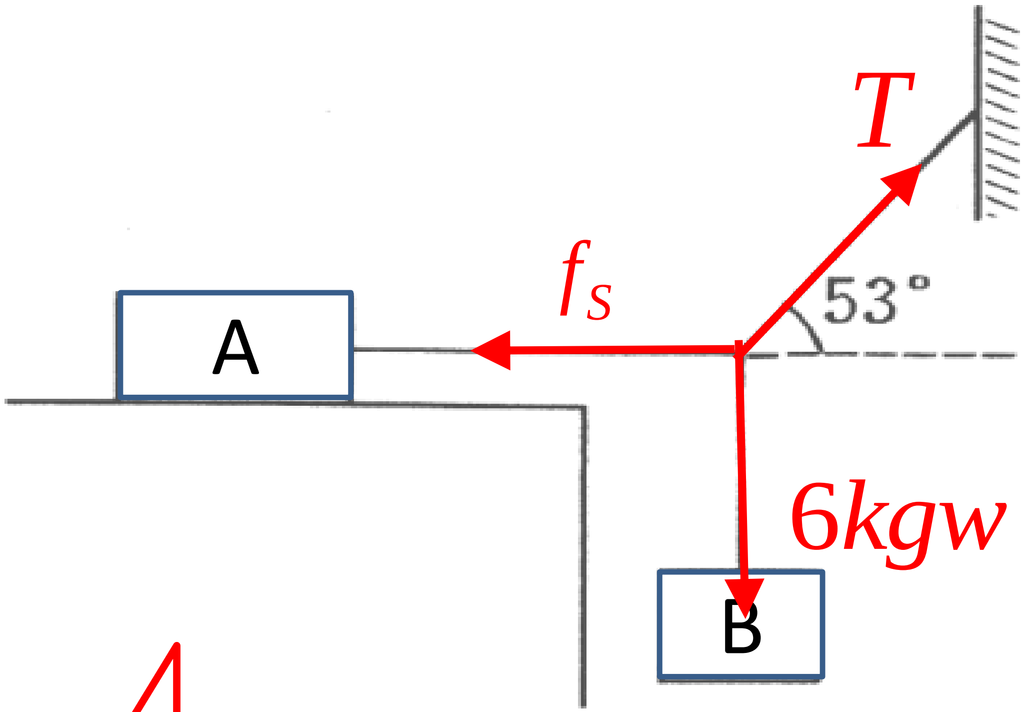
（1）若物體B重*6kgw*，此時作用於木塊之靜摩擦力為？

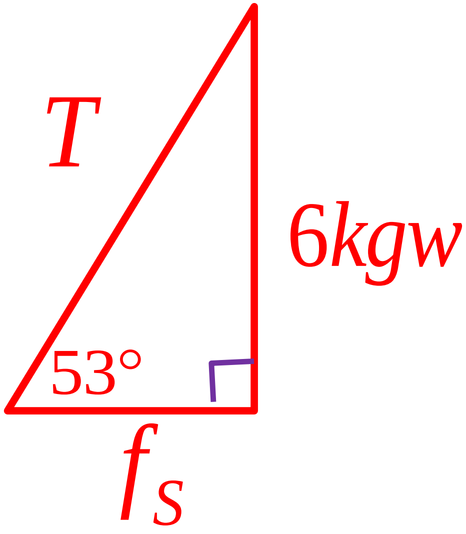
（2）欲保持靜止，物體B重量之最大值為？



[解析]

(1)

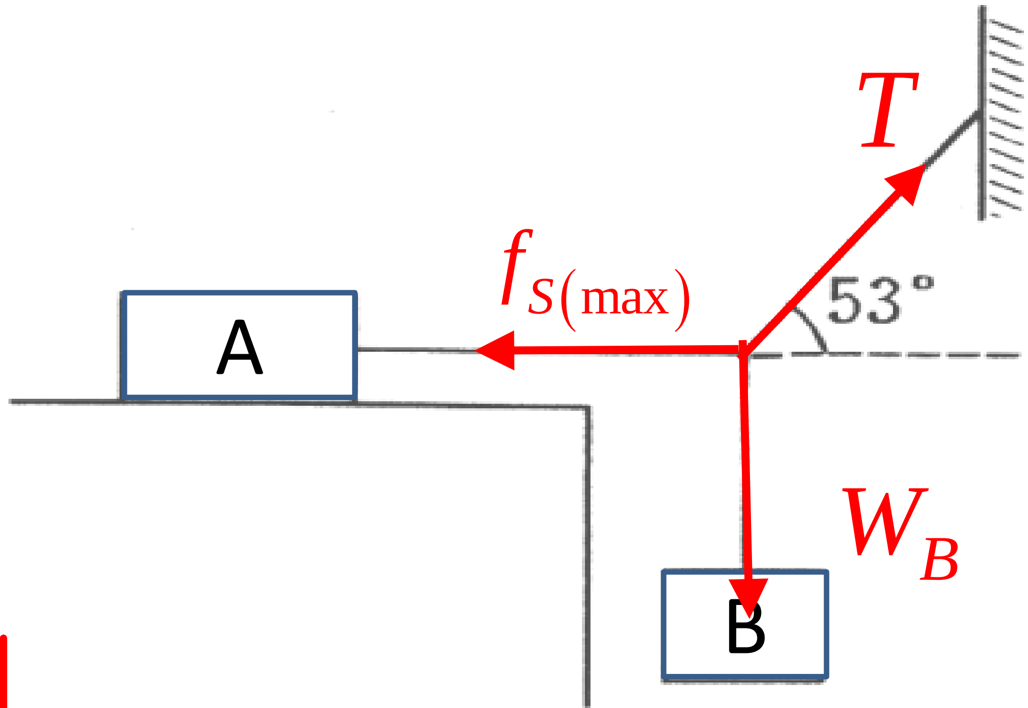


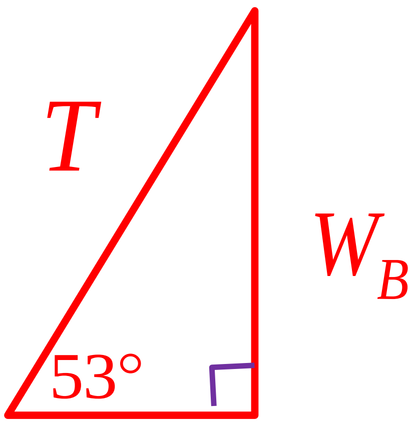


$$T=6\times\frac{5}{4}=\frac{15}{2} \quad f_s=6\times\frac{3}{4}=\frac{9}{2}$$

[解析]

(2)



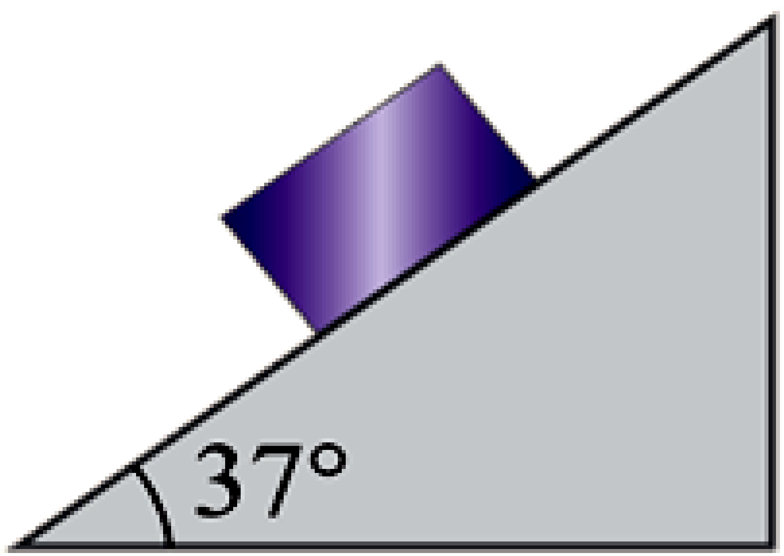


$f_{S(\max)} = 6kgw$

$$f_{S(\max)} = \mu_S N = 0.6 \times 10 = 6[kgw]$$

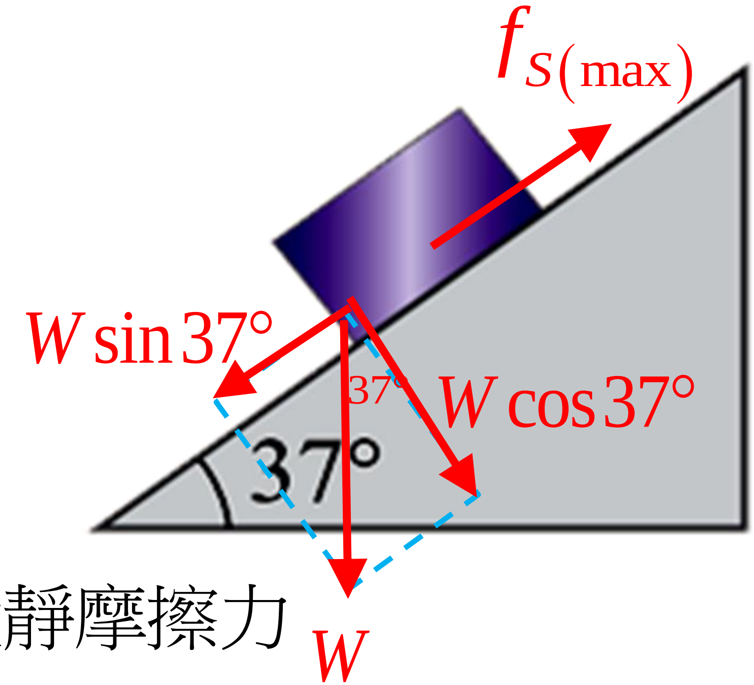
$$W_B = 6 \times \frac{4}{3} = 8$$

1. 將重量為*W*的物體靜置於與水平面成θ角的斜面上，當θ增至37度時，物體恰開始下滑，如圖(a)所示。則物體與斜面間的靜摩擦係數為________________。



▲圖(a)

[解析]



恰下滑時，摩擦力為最大靜摩擦力

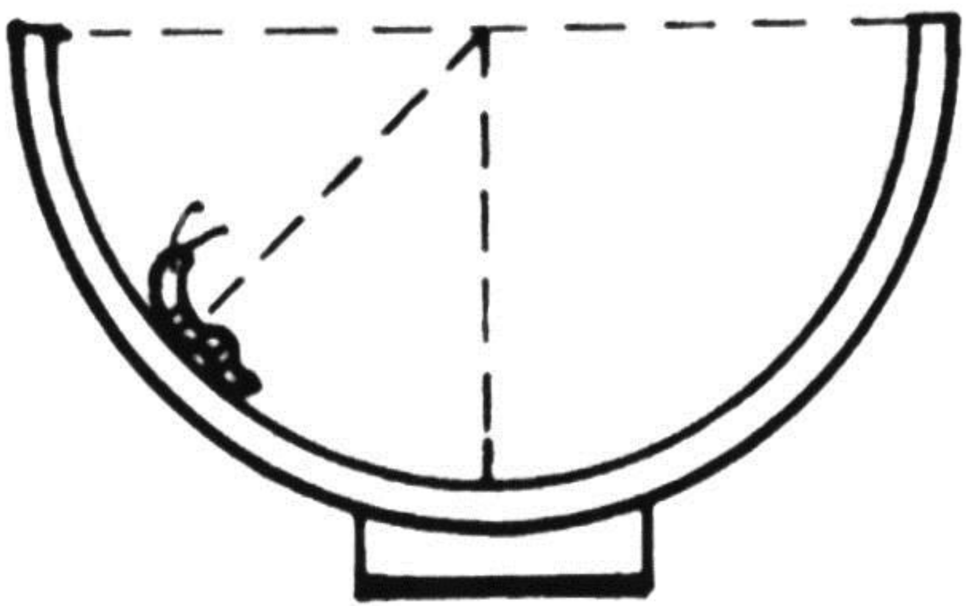
$$f_{S(\max)} = \mu_S W\cos 37°$$

合力 = 0

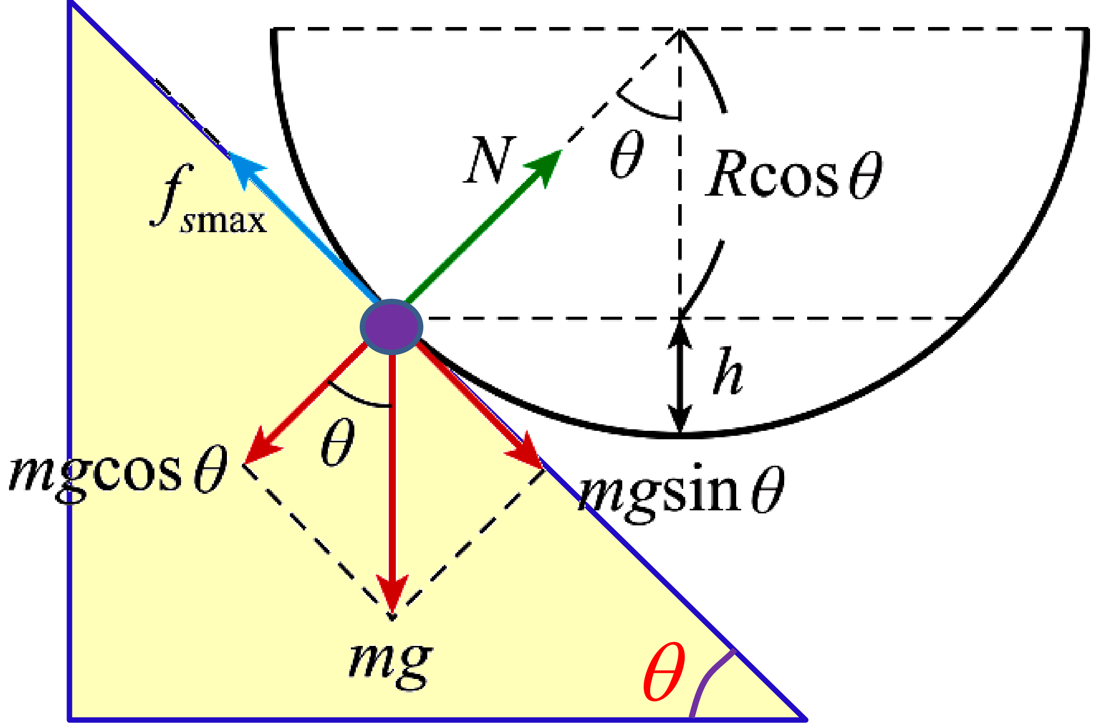
$$W\sin 37° = \mu_S W\cos 37° \quad \therefore \mu_s = \tan 37° = \frac{3}{4} = 0.75$$

靜摩擦係數等於恰滑動時斜面傾斜角正切值 $\mu_S = \tan\theta$

2.如圖一小蟲重量 mg，在半徑為R之碗壁沿壁上爬，若蟲與碗壁間摩擦係數為 $\sqrt{3}$ ，則：
（1）此蟲可爬升距碗底之最大鉛直高度為若干？
（2）小蟲達最大高度時碗壁對小蟲之作用力為若干？



[解析]



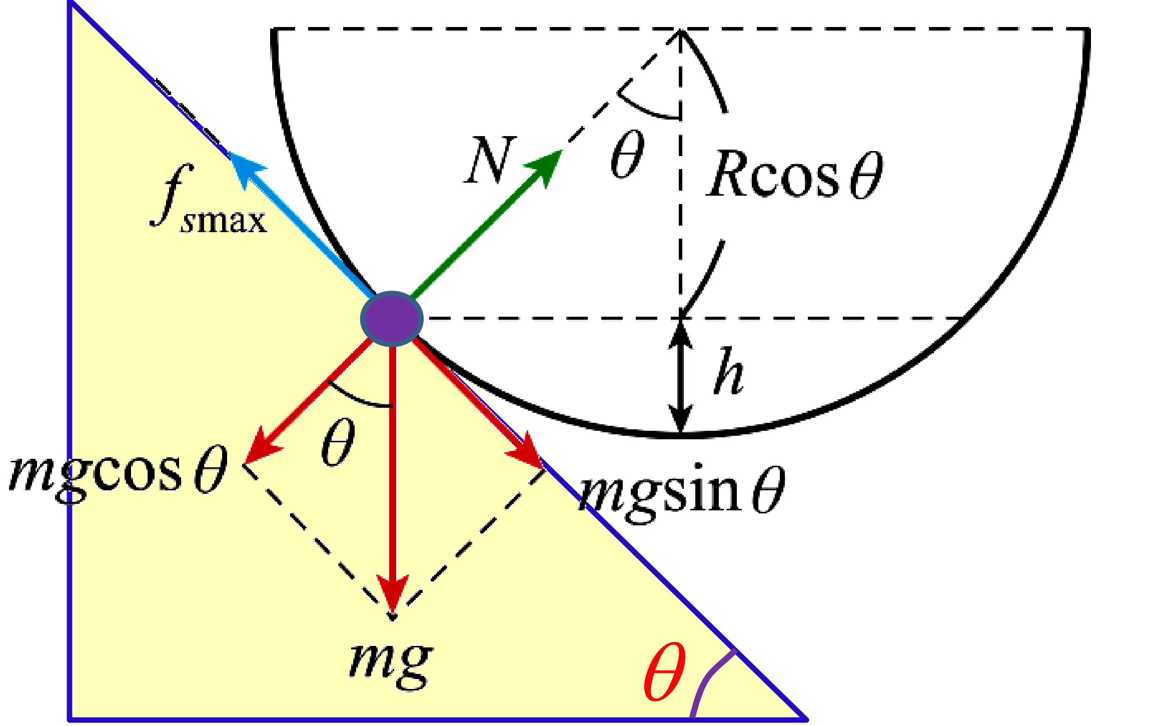
(1)

∵越高時,切線傾斜角θ越大,重力沿斜面分量(下滑力)越大
靜摩擦力越大

∴當最大高度時,摩擦力恰為最大靜摩擦力

[解析]

(1)

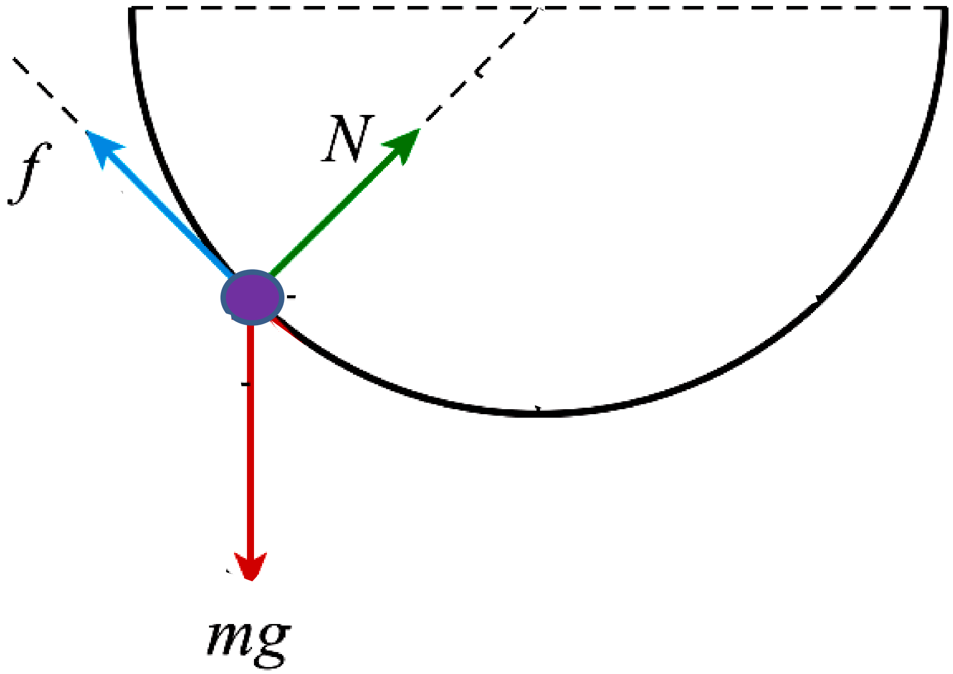


$$\begin{cases}\text{法線方向：} & g\cos\theta = N \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{①} \\ \text{切線方向：} & mg\sin\theta = f_{s\max} = \mu N \cdots\cdots\text{②}\end{cases}$$

$$\Rightarrow \frac{\text{②}}{\text{①}} \text{得} \tan\theta = \mu \Rightarrow \cos\theta = \frac{1}{\sqrt{1+\mu^2}}$$

$$\therefore h = R - R\cos\theta = R\left(1 - \frac{1}{\sqrt{1+\mu^2}}\right)$$

[解析]

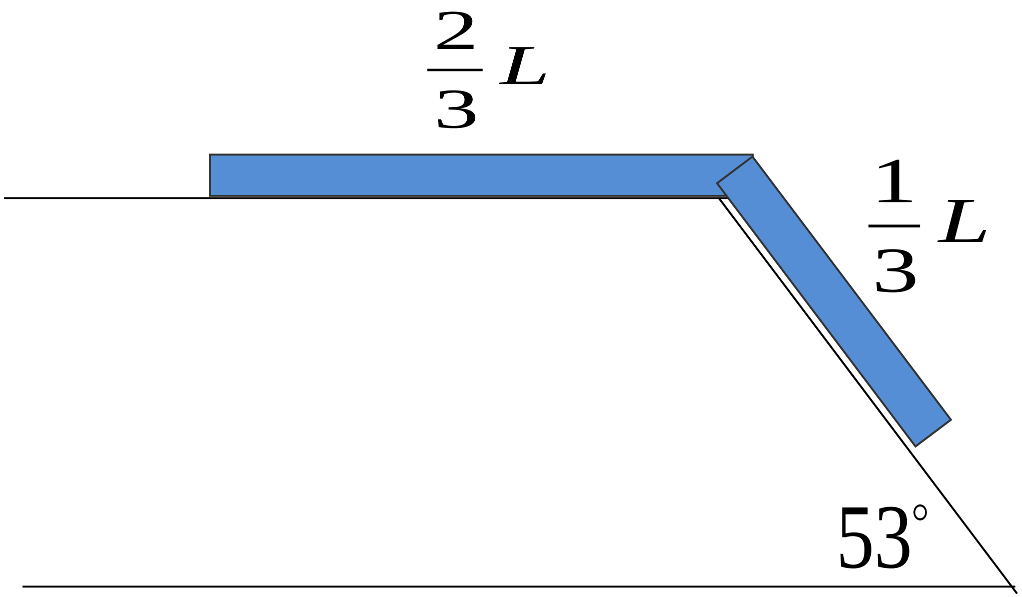


(2)靜止 $\rightarrow$ 合力=0 $\rightarrow \vec{f}+\vec{N}+m\vec{g}=0$

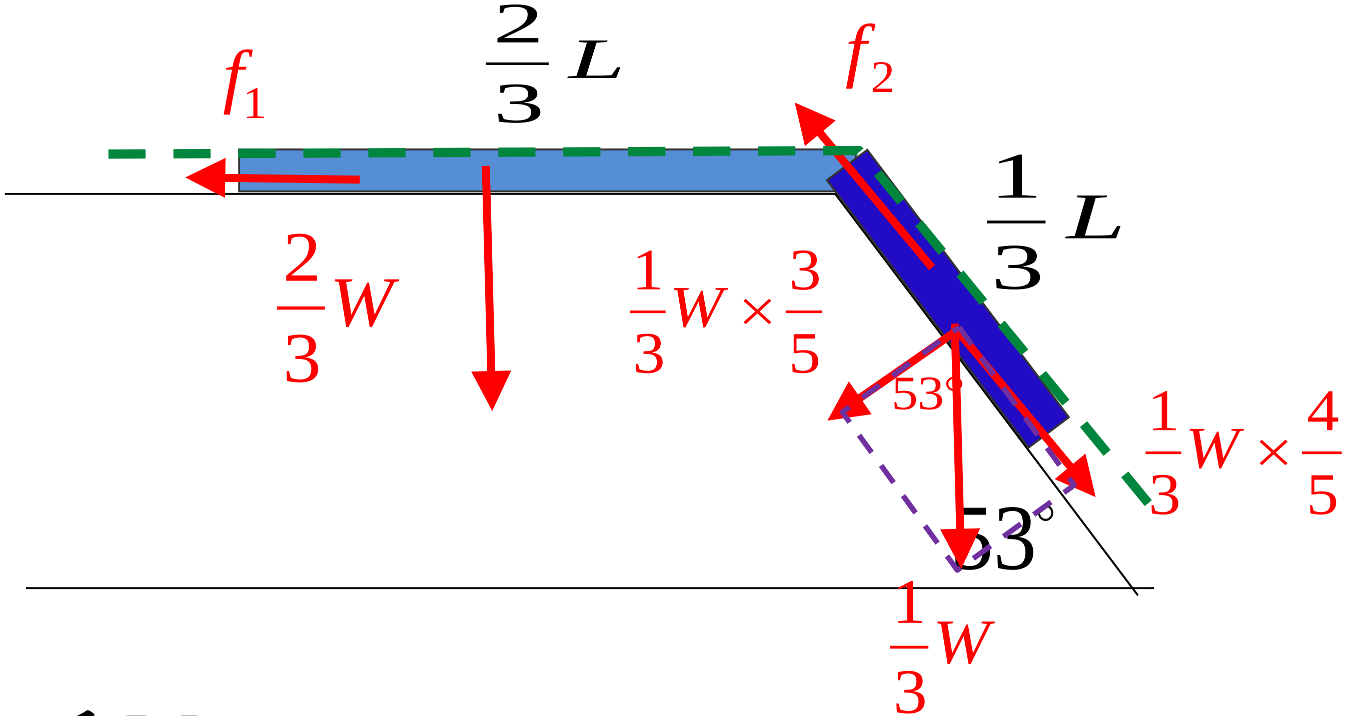
$\rightarrow$ 碗壁作用力 $=\vec{f}+\vec{N}=-m\vec{g}$

$\therefore$ 碗壁作用力大小等於重量

1.如圖，一均勻柔軟的細繩總長度L，$\frac{2}{3}L$ 在水平面上，$\frac{1}{3}L$ 在傾斜角53°的斜面上。若水平面及斜面的靜摩擦係數均為μ，此時細繩恰將下滑，則靜摩擦係數μ =？



[解析]



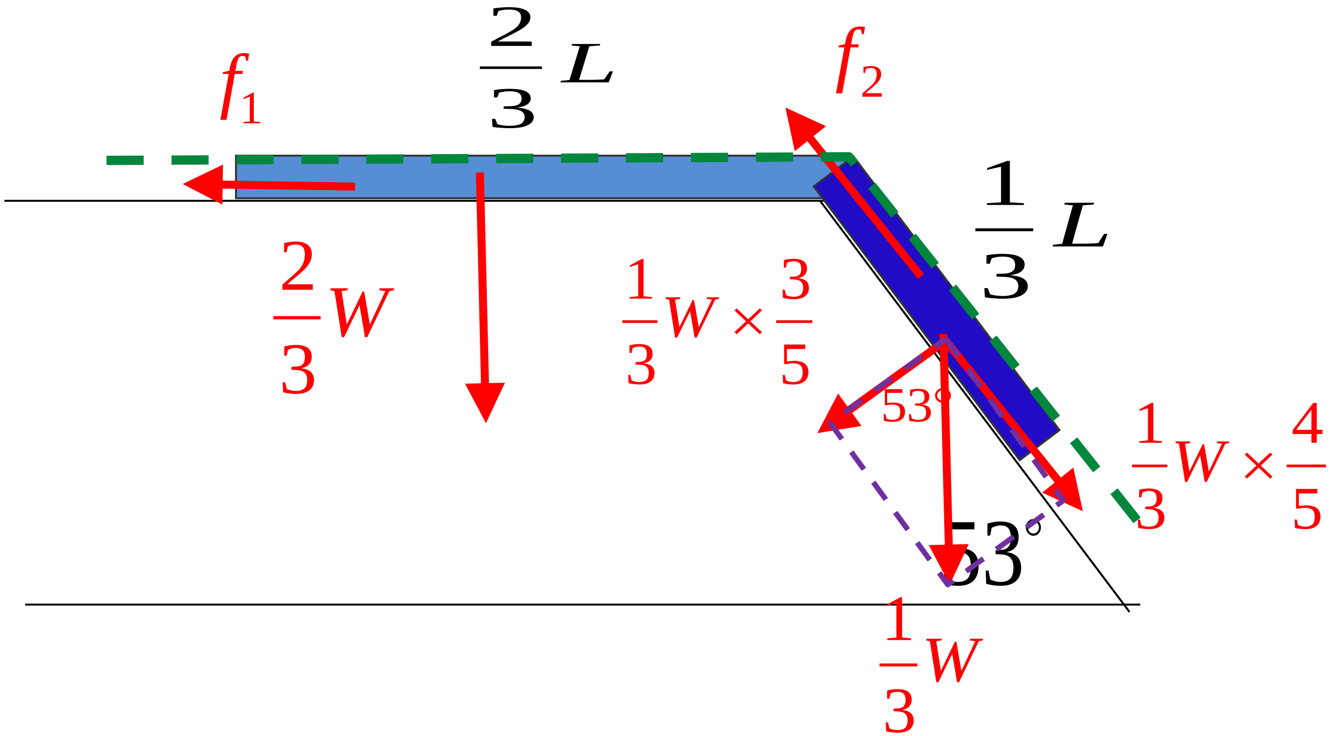
令繩重W

∵恰下滑∴摩擦力為最大靜摩擦力

$$\left[f_{S(\max)}=\mu N\right]$$

$$\therefore f_1=\mu\frac{2}{3}W \quad f_2=\mu\frac{1}{3}W\times\frac{3}{5}$$

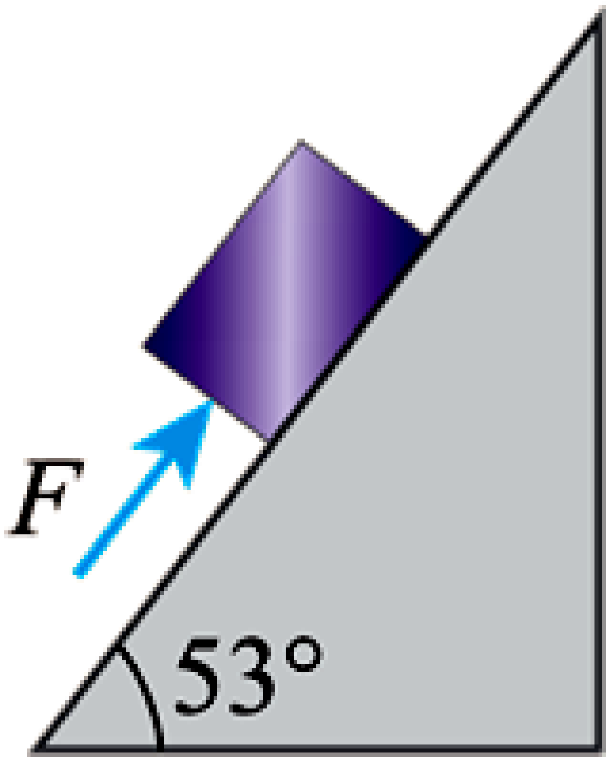
[解析]



恰下滑　合力=0

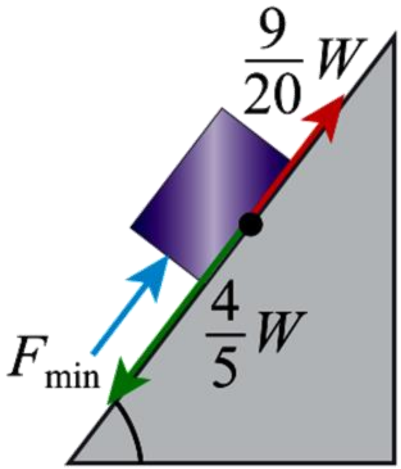
$$\frac{1}{3}W \times \frac{4}{5} = f_1 + f_2 = \mu \frac{2}{3}W + \mu \frac{1}{3}W \times \frac{3}{5} \quad \therefore \mu = \frac{4}{13}$$

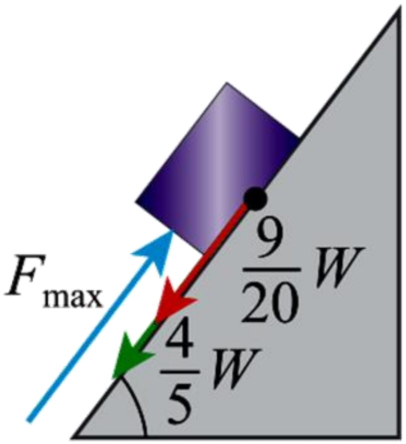
2. 現將θ增至53度，如圖(b)所示，欲使物體靜止於斜面上，沿斜面所施之力 *F*，則 *F* 的最小值為_____，最大值為_____。



▲圖 (b)

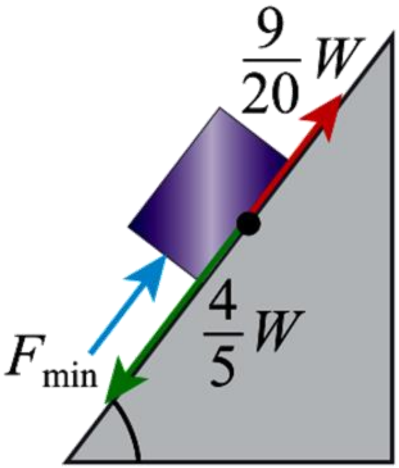
[解析]

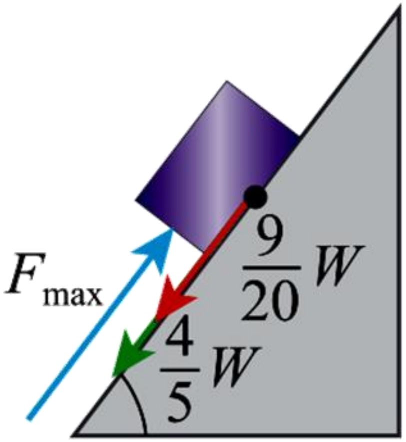




$$\begin{cases} \text{物體與斜面間的正向力} N = W\cos 53^\circ = \dfrac{3}{5}W \\ \Rightarrow \text{最大靜摩擦力} f_{s\max} = \mu_s N = \dfrac{3}{4} \times \dfrac{3}{5}W = \dfrac{9}{20}W \\ \text{物體的下滑力} = W\sin 53^\circ = \dfrac{4}{5}W \end{cases}$$

[解析]





$$\Rightarrow \begin{cases} F_{\min} = \frac{4}{5}W - \frac{9}{20}W \Rightarrow F_{\min} = \frac{7}{20}W \\ F_{\max} = \frac{4}{5}W + \frac{9}{20}W = \frac{5}{4}W \end{cases}$$

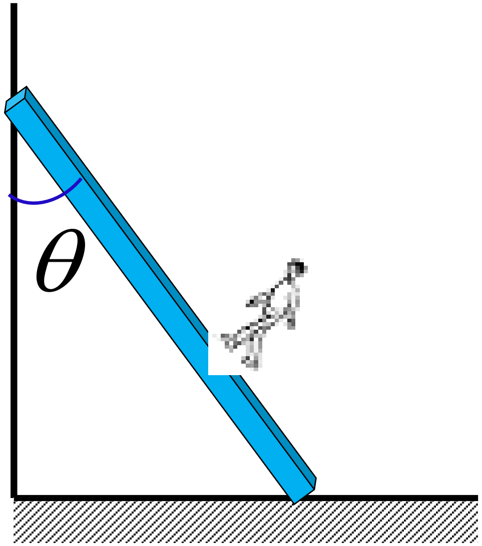
一梯子上端斜靠鉛直光滑牆面，梯與牆之夾角為θ，梯的下端在水平地面上。梯長為L，其質量不計。重量為W的人，直立於梯下端量起 $\frac{L}{3}$ 處，此系統平衡如圖示，試問：

(A) 牆施於梯的力量值為？

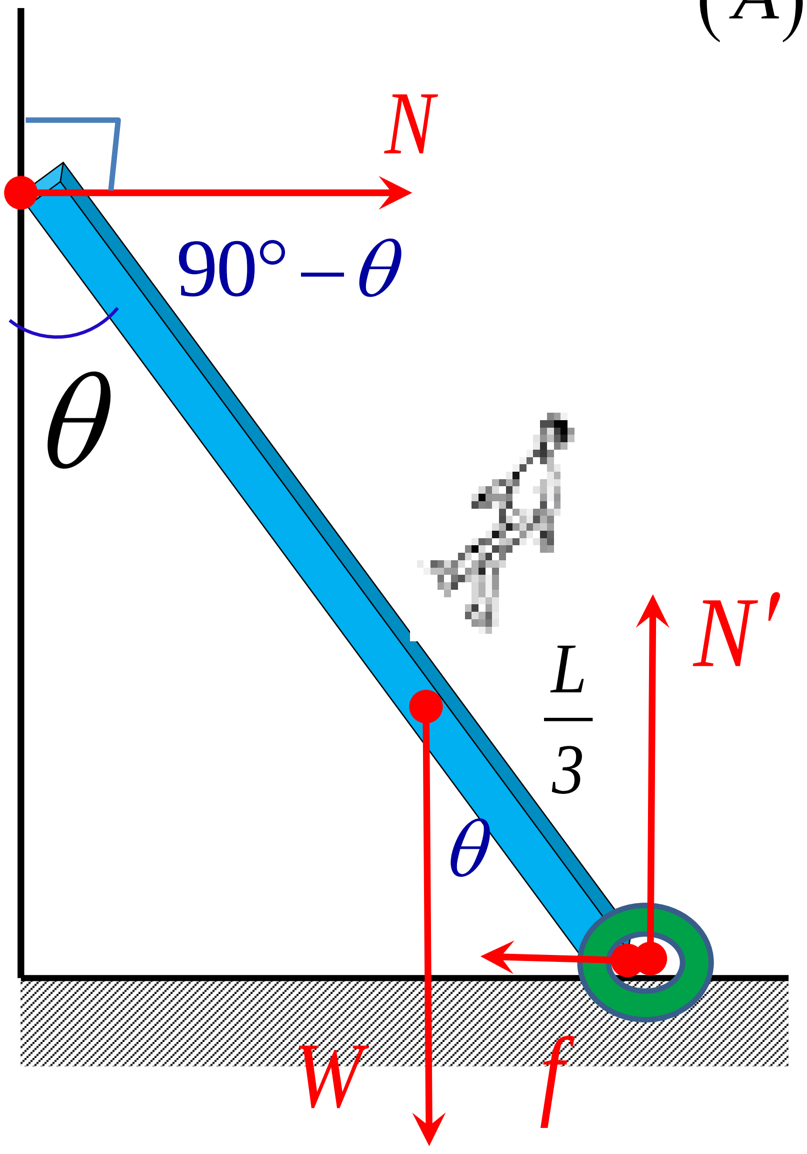
(B) 地面施於梯的力量值為？

(C) 地面作用於梯之力的方向與地面所夾的角度的正切值為？

(D) 地面與梯子間的靜摩擦係數至少為何？



[解析]



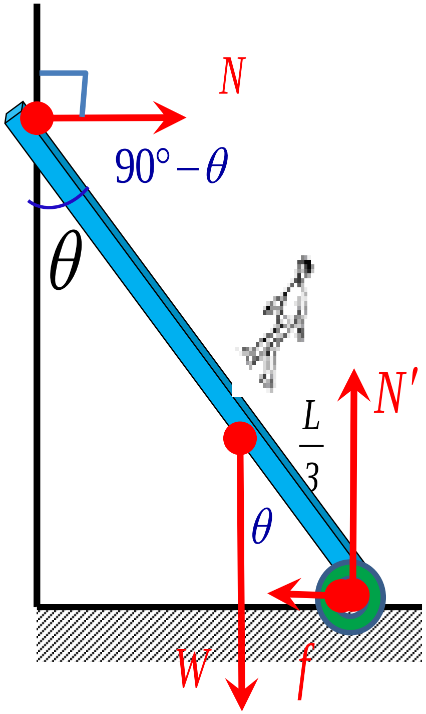
(A)

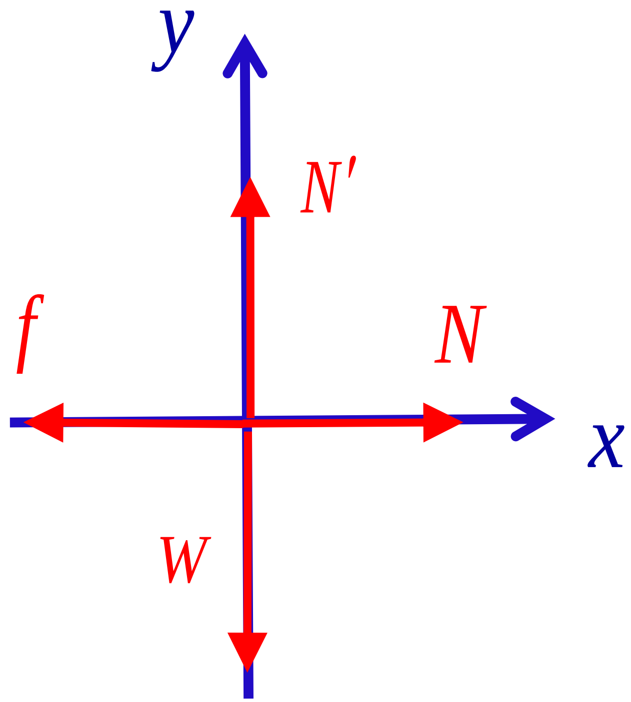
以觸地端為轉軸　合力矩 $= 0$

$$\frac{\ell}{3} \times W \times \sin\theta = L \times N \times \sin(90° - \theta)$$

$$\therefore N = \frac{W}{3}\tan\theta$$

[解析]





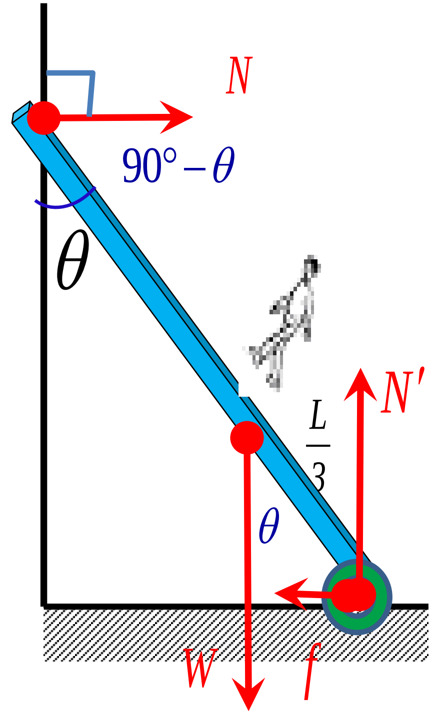
$$(B)\text{合力}=0\ \begin{cases} x: f = N = \dfrac{W}{3}\tan\theta \\ y: N' = W \end{cases}$$

$$(C)\therefore \tan\phi = \frac{N'}{f} = \frac{W}{\dfrac{W}{3}\tan\theta} = \frac{3}{\tan\theta}$$

[解析]

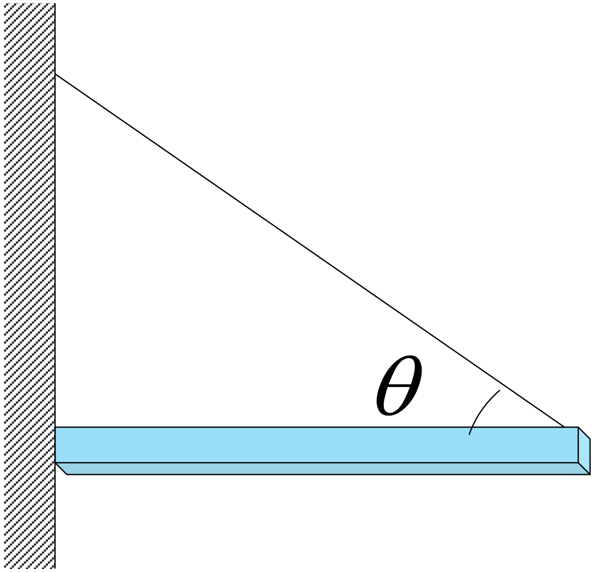
(D)靜止不滑動

$$f \le f_{s(\max)} \rightarrow \frac{W}{3}\tan\theta \le \mu_s W \therefore \mu_s \ge \frac{\tan\theta}{3}$$

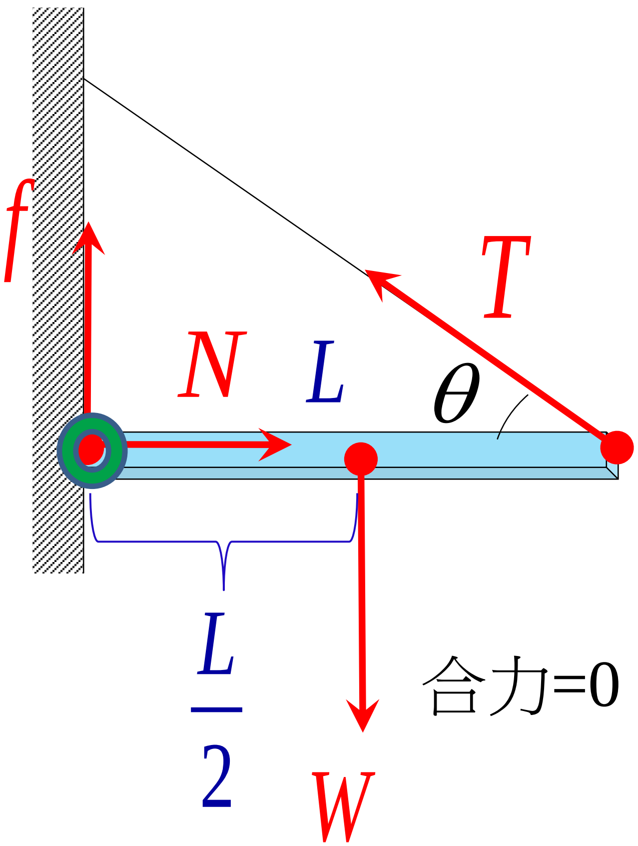


第127頁

1.圖中均勻長棒的一端垂直頂在一鉛直牆上，另一端以輕繩繫於牆，棒與牆的靜摩擦係數為 $\frac{1}{\sqrt{3}}$，欲使棒保持水平，繩與棒夾角θ的最大值為何？



[解析]



以觸牆端為轉軸　合力矩 $=0$

$$L \times T \times \sin\theta = W \times \frac{L}{2} \quad \therefore T = \frac{W}{2\sin\theta}$$

合力=0 $\begin{cases} 水平: N = T\cos\theta = \dfrac{W}{2\tan\theta} \\ 鉛直: f + T\sin\theta = W \rightarrow f = W - T\sin\theta = \dfrac{W}{2} \end{cases}$

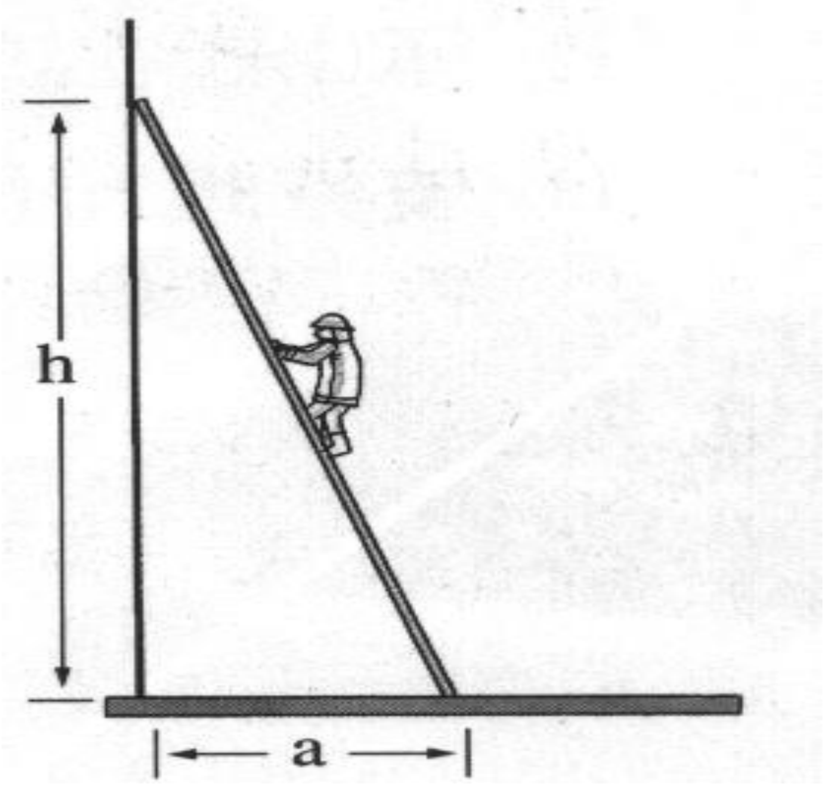
$$\therefore f \le f_{S(\max)} = \mu_S N = \frac{\mu_S W}{2\tan\theta} \rightarrow \frac{W}{2} \le \frac{\mu_S W}{2\tan\theta}$$

$$\rightarrow \tan\theta \le \mu_S = \frac{1}{\sqrt{3}} \therefore \theta \le 30^\circ$$

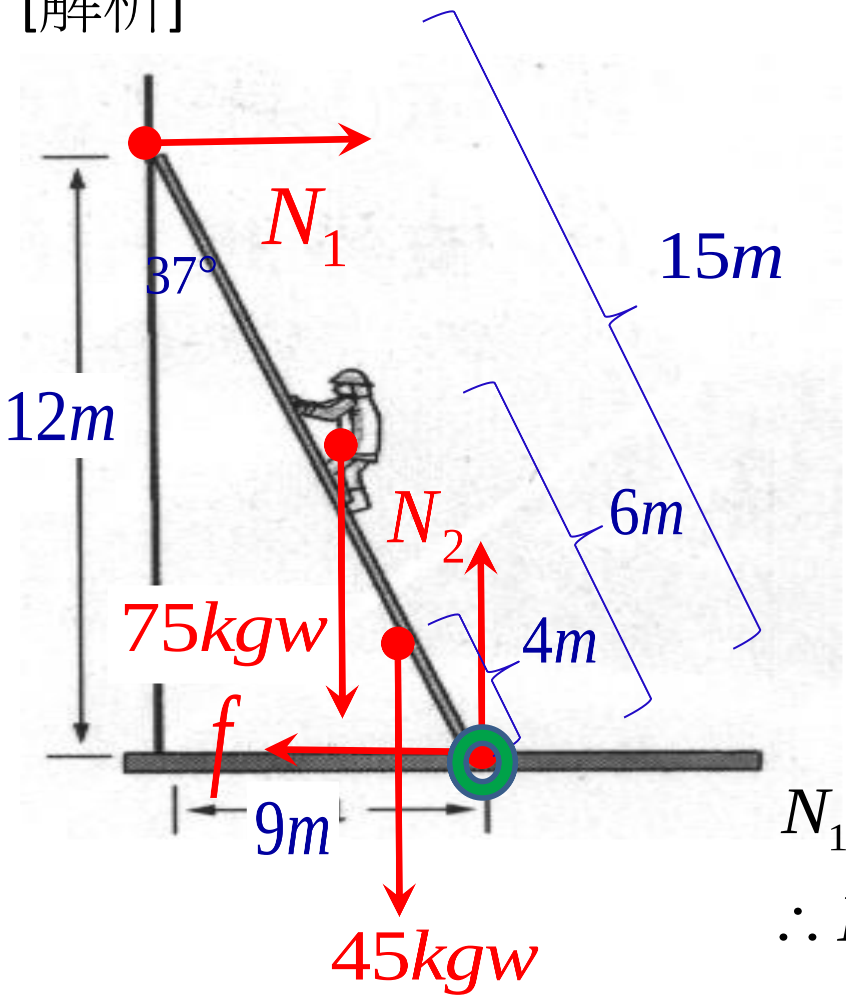
2.一梯長15 m、重45 kgw，靠在牆和地面上，其頂端離地面*h=12m*，梯子的重心與其底端距離為4 m；若牆面光滑，一重75 kgw的消防隊員立於梯子6 m處(沿梯自下端量起)成平衡狀態，試問：

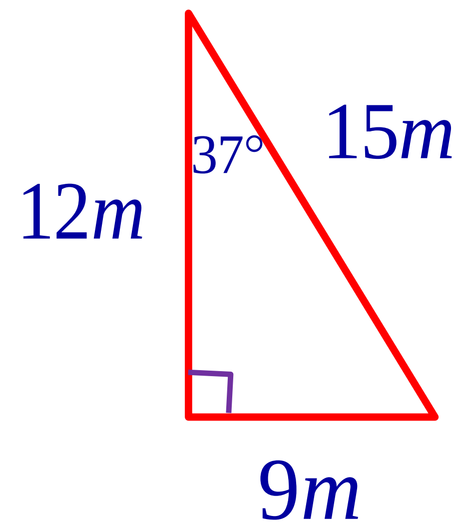
（*a*）梯子和牆面及地面的作用力各為若干？

（*b*）若梯子與地面間的靜摩擦係數為0.5，則此消防隊員最高可爬到哪裏？



[解析]　(1)





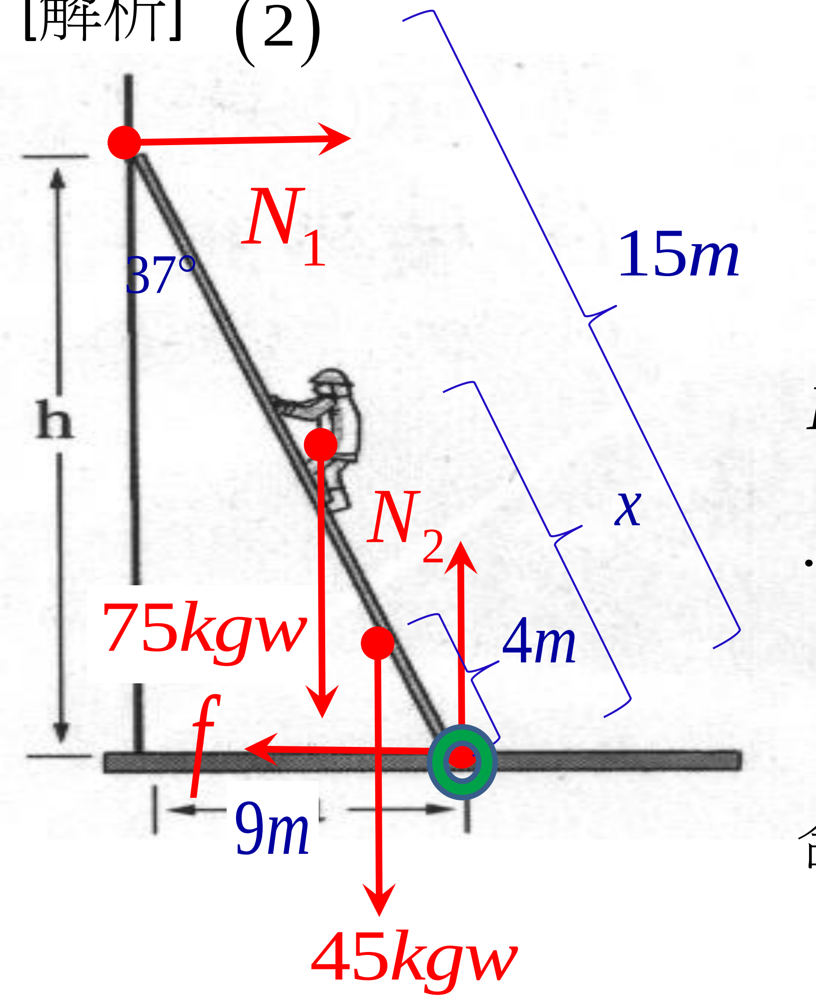
以觸地端為轉軸　合力矩＝0

$$N_1 \times 12 = 6 \times 75 \times \sin 37° + 4 \times 45 \times \sin 37°$$

$$\therefore N_1 = 31.5[kgw]$$

$$合力=0 \begin{cases} 水平: f = N_1 = 31.5[kgw] \\ 鉛直: N_2 = 75 + 45 = 120[kgw] \end{cases}$$

[解析] (2)



令最高距底端 x

以觸地端為轉軸　合力矩 = 0

$$N_1 \times 12 = x \times 75 \times \sin 37^\circ + 4 \times 45 \times \sin 37^\circ$$

$$\therefore N_1 = \frac{15}{4}x + 9$$

$$合力=0 \begin{cases} 水平: f = N_1 = \dfrac{15}{4}x + 9 \\ 鉛直: N_2 = 75 + 45 = 120\left[kgw\right] \end{cases}$$

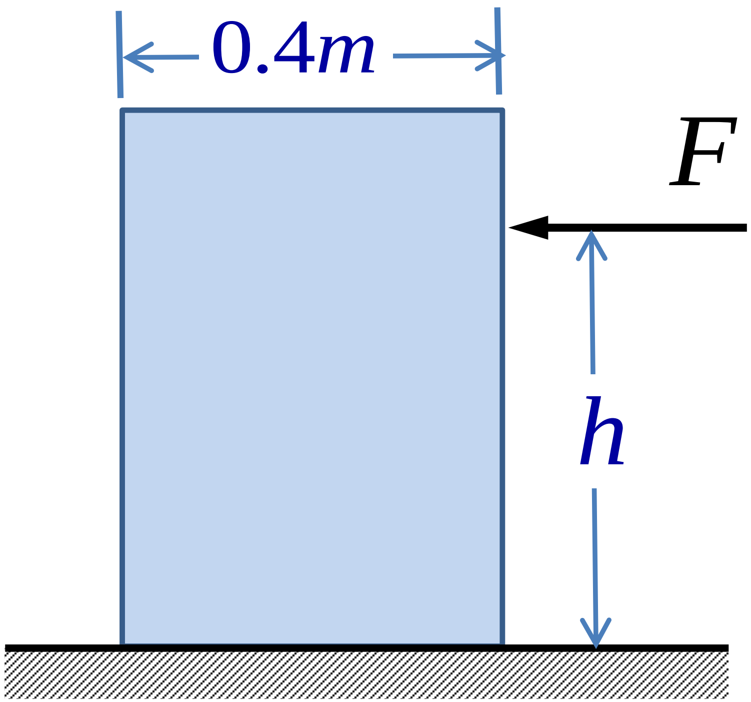
$$\because f \le f_{S(\max)} = \mu_S N_2 = 0.5 \times 120 = 60$$

$$\therefore \frac{15}{4}x + 9 \le 60 \rightarrow x \le \frac{68}{5}$$

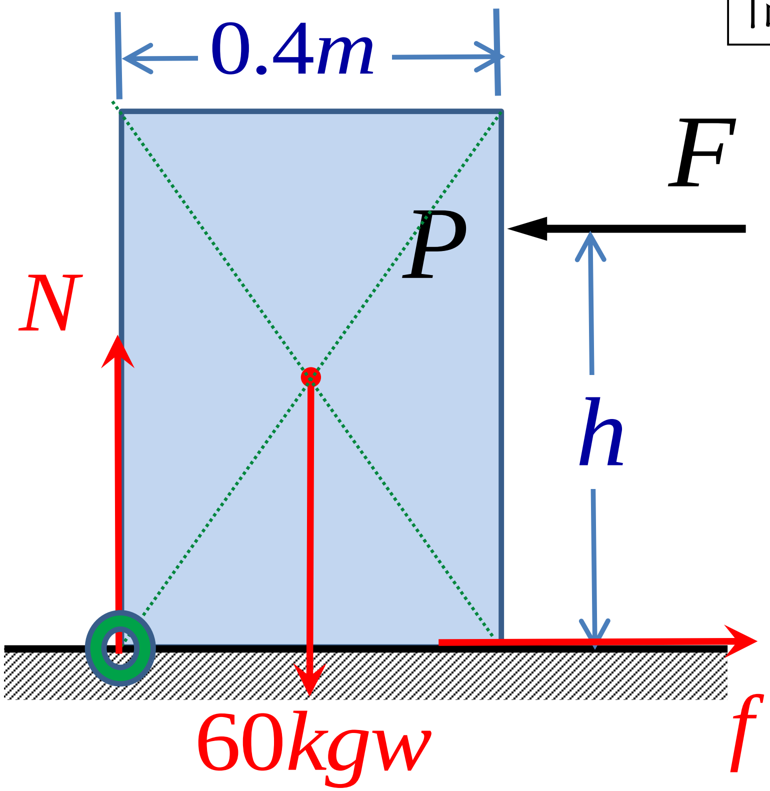
1.如圖，一木箱置於水平面上，已知木箱重60公斤重，箱與地面間的靜摩擦係數為0.2。今施一水平力推此木箱，木箱恰要同時發生移動與翻倒現象，試問：

(A)地面對木箱的正向力量值為？ (B)地面的摩擦力量值為？

(C)水平力的量值為？ (D) P點離地的高度為？



[解析]



恰移動時，摩擦力恰為最大靜摩擦力

$$\begin{cases} 水平: F = f \\ 鉛直: N = 60[kgw] \end{cases}$$

$$\because f = f_{S(\max)} = \mu_S N = 0.2 \times 60 = 12$$

$$\therefore F = 12[kgw]$$

恰翻轉時，正向力恰通過轉軸

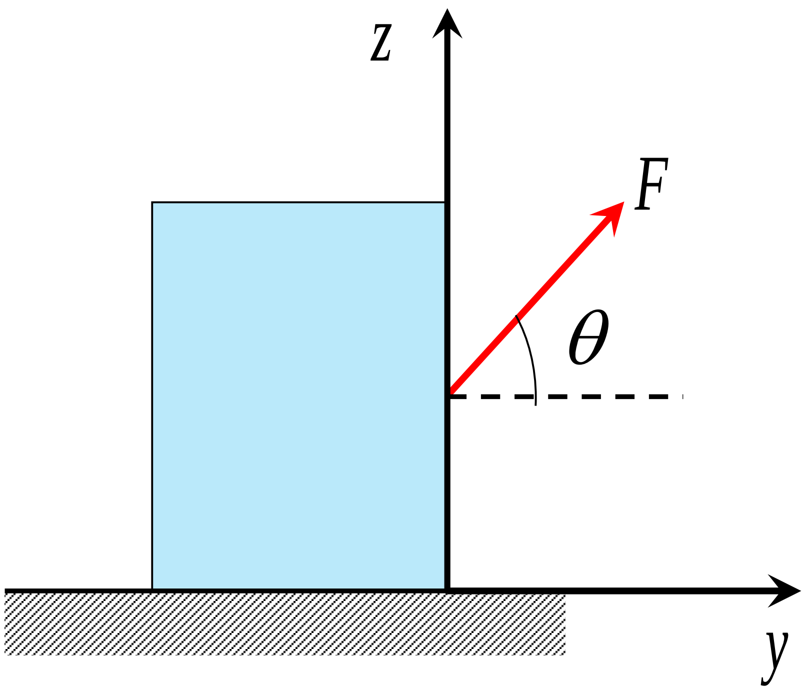
恰翻轉時　合力矩 = 0

$$F \times h = 60 \times 0.2$$

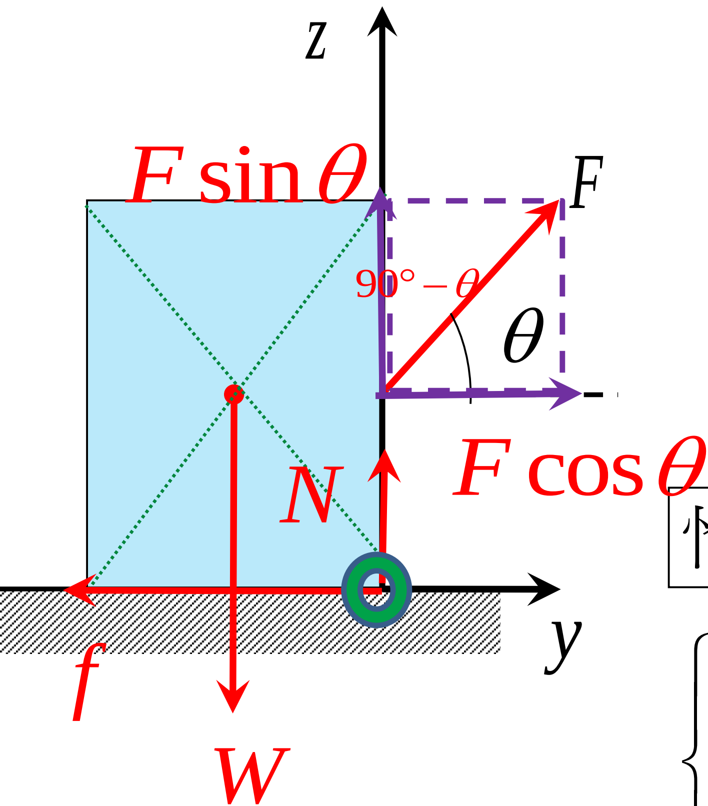
$$\therefore h = \frac{12}{12} = 1[m]$$

第128頁

2.水平桌面上有一邊長為L的正立方體，今在此立方體的某垂直面($x-z$平面)的正中央繫一繩，以此繩拉立方體，繩在$y-z$平面且與水平的y方向成θ角，如圖所示。當拉力F逐漸增大時，發現在立方體開始滑動的同時，亦開始以x軸為轉軸發生轉動。設桌面與立方體間的靜摩擦係數為μ_s，則μ_s之值為？



[解析]



$\boxed{\text{恰翻轉時，正向力恰通過轉軸}}$

恰翻轉時　合力矩 = 0

$$\frac{L}{2}\times F\times\sin\left(90^\circ-\theta\right)=W\times\frac{L}{2}$$

$$\therefore F=\frac{W}{\cos\theta}$$

$\boxed{\text{恰移動時，摩擦力恰為最大靜摩擦力}}$

$$\begin{cases}\text{水平}: f=F\cos\theta=\dfrac{W}{\cos\theta}\cos\theta=W\\ \text{鉛直}: N+F\sin\theta=W\rightarrow N=W\left(1-\tan\theta\right)\end{cases}$$

$$\because f=f_{S(\max)}=\mu_S N=\mu_S W\left(1-\tan\theta\right)=W$$

$$\therefore \mu_S=\frac{1}{\left(1-\tan\theta\right)}$$